Physical acoustics – Numerical examples with guided waves

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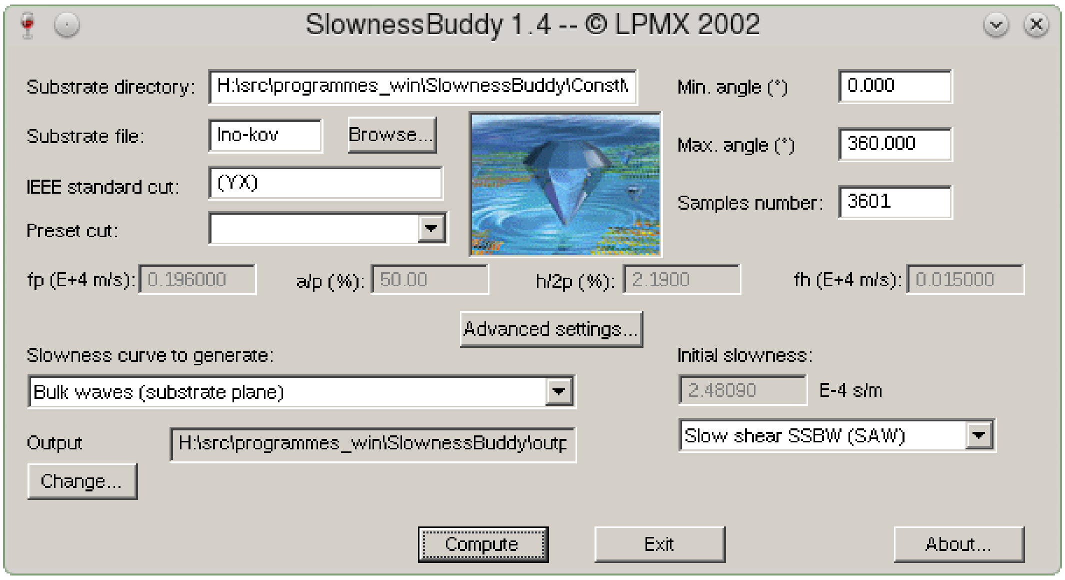
1 SlownessBuddy

SlownessBuddy is a Windows program that can only do one thing: compute slowness curves, for both bulk and surface acoustic (elastic) waves in (piezoelectric) solids. The program uses a plane wave formulation:

- Christofell's equation for bulk elastic waves,
- Surface boundary condition determinant minimization for SAW,
- Finite element method/boundary element method combination for thick SAW transducers.



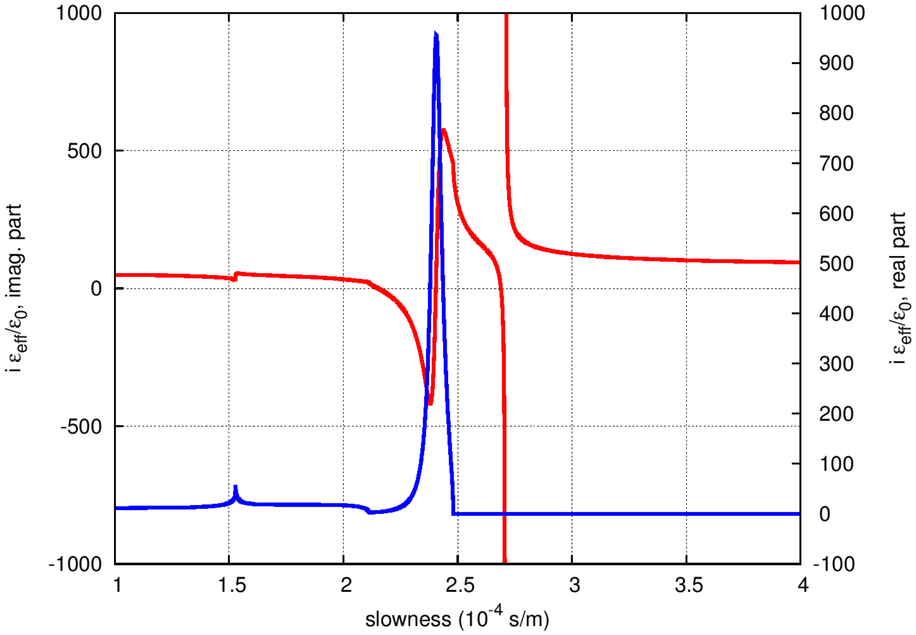
2 ... and a Julia library for bulk and surface waves

Julia is a language similar to Python or Matlab, but more modern :-) Source code (under development) available upon request. It is used for Brillouin light scattering numerical simualtion as well.

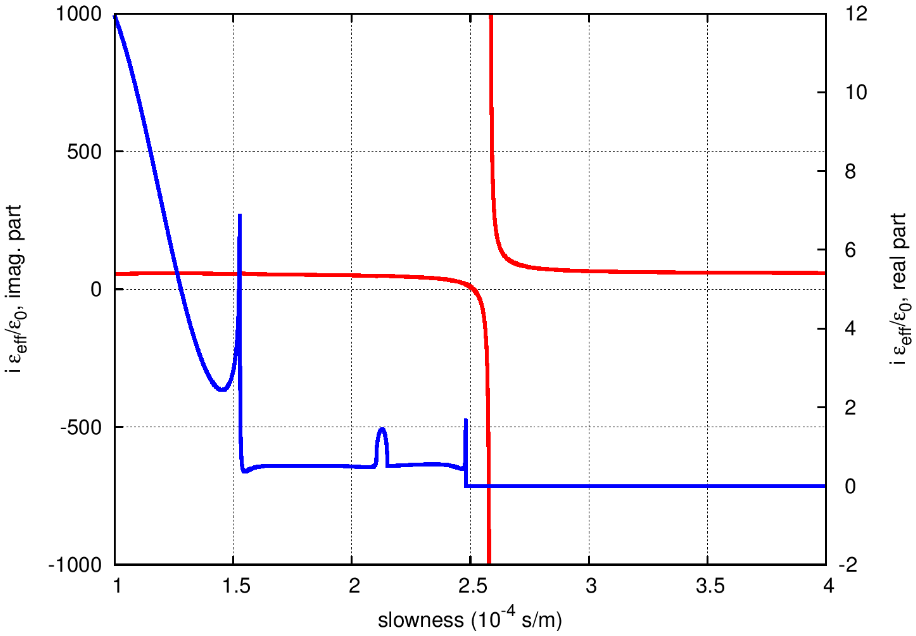
3 Effective surface permittivity

Consider the following 3 cuts of lithium niobate in IEEE notation: (YX), (YXl)/128, (YXl)/36. Can you understand from the effective permittivity plots which cut is (from the point of view of SAW technology):

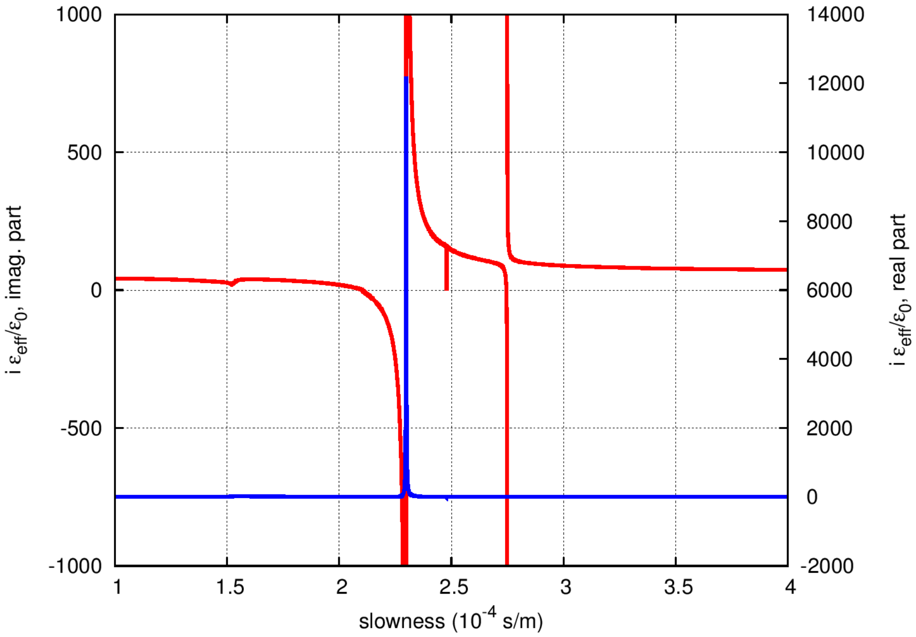
1. Ideal for lossless SAW generation?
2. Not worth anything?
3. Quite interesting for leaky-SAW (PSAW) generation?



(YX)



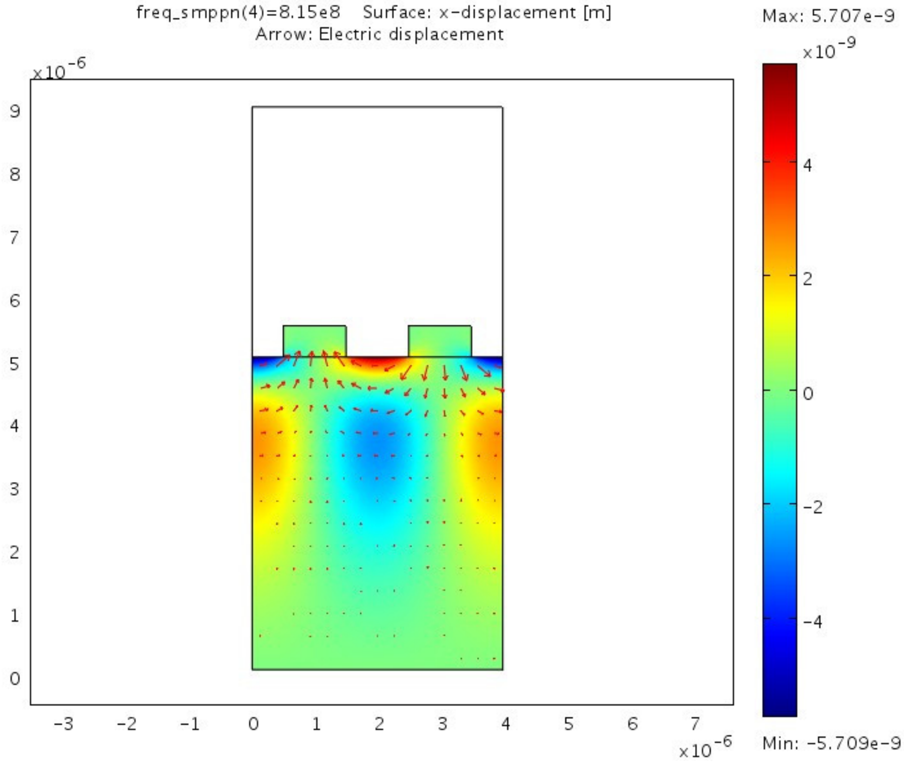
(YXl)/128



(YXl)/36

4 Brute force numerical simulation of a thick SAW periodic transducer

Let us consider a substrate of lithium niobate. An interdigital transducer with pitch $p = 2$ μm is has a wavelength at resonance of $\lambda = 4$ μm. Can we perform a brute force finite element simulation to obtain the distribution of the displacements of the generated surface acoustic wave?



5 And Lord Rayleigh found the surface elastic wave in 1885...

Let us consider the surface of an isotropic solid with velocities c_L and c_S for longitudinal and shear waves. Propagation is along direction x_1 and x_2 is orthogonal to the surface (and pointing outward of the solid). We want to find an analytical formula for the slowness s_1 of the Rayleigh wave.

1. Justify that there are two partial waves in the solid, one purely longitudinal and one purely shear. Justify that the slowness vector for the L partial is $s_1(1, i\beta_L)^T$ with $\beta_L = \sqrt{1 - c_L^{-2}/s_1^2}$. Justify that the slowness vector for the S partial is $s_1(1, i\beta_S)^T$ with $\beta_S = \sqrt{1 - c_S^{-2}/s_1^2}$.

2. Justify that the polarization (displacement vector) for the L partial wave is proportional to $(1, i\beta_L)^T$. Justify that the polarization (displacement vector) for the S partial wave is proportional to $(-i\beta_S, 1)^T$.

3. Obtain the strains S_1, S_2, and S_6 at the surface. Observe that the other strains are zero. Nota: first express the total displacement in the solid as a surperposition of the two partial waves with amplitudes a_L and a_S.

4. Show that the boundary condition at the surface is $T_2 = 0$ and $T_6 = 0$. Then express this boundary condition as the matrix relation

$$\begin{pmatrix} i(1+\beta_S^2) & 2\beta_S \\ 2\beta_L & -i(1+\beta_S^2) \end{pmatrix} \begin{pmatrix} a_L \\ a_S \end{pmatrix} = 0 \quad (1)$$

5. Conclude that the secular equation for the Rayleigh SAW is

$$(1+\beta_S^2)^2 - 4\beta_S\beta_L = 0 \quad (2)$$