Physical acoustics – practical class

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The version of Comsol Multiphysics that is available in UFR-ST room 217B is 3.5, circa 2008 (old but stable!). It is a class-kit version (slightly limited in terms of memory, but it is enough for our purposes). If you have a more recent licence with you, you can certainly use it.

Sections 1–3 should be feasible within the time limit, even if you have never performed finite element simulations before. Further sections are for the braves!

## 1   Diffusion of an acoustic wave by an air bubble

We want to model the diffusion of an acoustic wave in water by an air bubble. The acoustic wave is generated by an immersed ultrasonic transducer. Water is modeled by $\rho = 1000$ kg/m^3 and $c_s = 1480$ m/s (acoustic velocity), air by $\rho = 1.25$ kg/m^3 and $c_s = 343$ m/s. For simplicity, we consider a 2D model. The transducer is modeled by a rectangular box of height 12 cm and thickness 1 cm. The air bubble is modeled by a circle with radius 5 mm.

1. Open Comsol 3.5, choose "2D", mode "Acoustics/Time-harmonic analysis". Create a disk of water with a radius of 10 cm. Add to the left the transducer and an air bubble to the right.

2. Define boundary conditions for the transducer: all faces are continuous for pressure but the right-hand face applies a pressure of 1 Pa.

3. Define exterior boundary conditions as "radiation/cylindrical wave". Can you explain what they are for?

4. Mesh, solve and observe the diffraction for frequencies spanning 10 to 150 kHz. Find a mesh that ensures a correct computation for all frequencies (describe the characteristics of such a mesh).

2   Phononic crystal

We now consider a periodic arrangement of 5 by 8 steel rods in water, as depicted in the figure. The dimensions of the transducer and of the exterior circle are identical to the previous problem. The steel rods have a diameter of 5 mm. The spatial period is 15 mm.

1. Open Comsol 3.5, choose "2D", "Multiphysics" and add the modes "Acoustics/Time-harmonic analysis" and "Structural Mechanics/Plane strain". Select mode "Acoustics" Create the drawing of the figure.
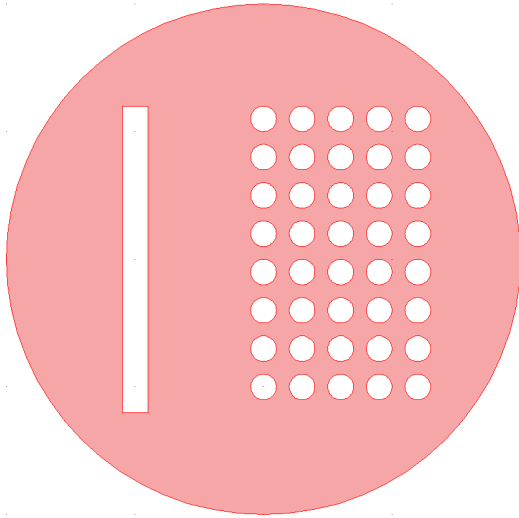
2. We first represent steel with an equivalent fluid, using its mass density (7850 kg/m^3) and the longitudinal velocity $\sqrt{c_{11}/\rho}$ with $c_{11} = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}$ ($E = 200$ GPa and $\nu = 0.33$). Define the constants in every region of the computation domain.

3. Define boundary conditions (note: they are identical to those of the previous problem).

4. Mesh, solve and observe the result for frequencies spanning 10 to 120 kHz. Find a mesh that ensures a correct computation for all frequencies. Note: at 85 kHz you should observe a *phononic band gap*. Comment the various effects you observe as a function of frequency (reflection, transmission, diffraction, resonance, etc.). In particular, is there a frequency bandwith (not containing 85 kHz) inside which you observe clearly a phononic band gap? Save the computation as an .mph file.

3 Acoustic / elastic coupling

We consider exactly the same problem as before, but now we represent steel as an elastic solid medium, which is indeed closer to reality.

1. Open the previous .mph file in order to recover the drawing and definitions.

2. Identify in the interface the different regions of "acoustic" and "elastic" type (check or uncheck "Active in this domain" in "Subdomain settings"). Enter the material constants for steel.

3. In the "Plane strain" module, enter the following boundary conditions arounf steel rods: "Fx=nx*p" and "Fy=ny*p". What is the physical meaning of such an expression?

4. In the "Acoustics" module, enter the following boundary conditions around the steel rods: normal acceleration
"an=(u*nx+v*ny)*(2*3.141592*freq_aco)^2". What is the physical meaning of such an expression?

5. Mesh, solve and observe the result for frequencies spanning 10 to 120 kHz. Find a mesh that ensures a correct computation for all frequencies. How different is the result from the purely acoustic computation? Observe displacements in the rods.

4 phononic crystal cavity and waveguide

1. Starting from Section 3, try to create a defect cavity (e.g. by removing one rod). Can you excite cavity modes trapped by the phononic band gap?

2. Now create a waveguide by removing a full line of rods. Can you observe transmission within the phononic band gap?

5 Brute force numerical simulation of a thick SAW periodic transducer

Let us consider a substrate of lithium niobate. An interdigital transducer with pitch $p = 2\ \mu$m is has a wavelength at resonance of $\lambda = 4\ \mu$m. Can we perform a brute force finite element simulation to obtain the distribution of the displacements of the generated surface acoustic wave?
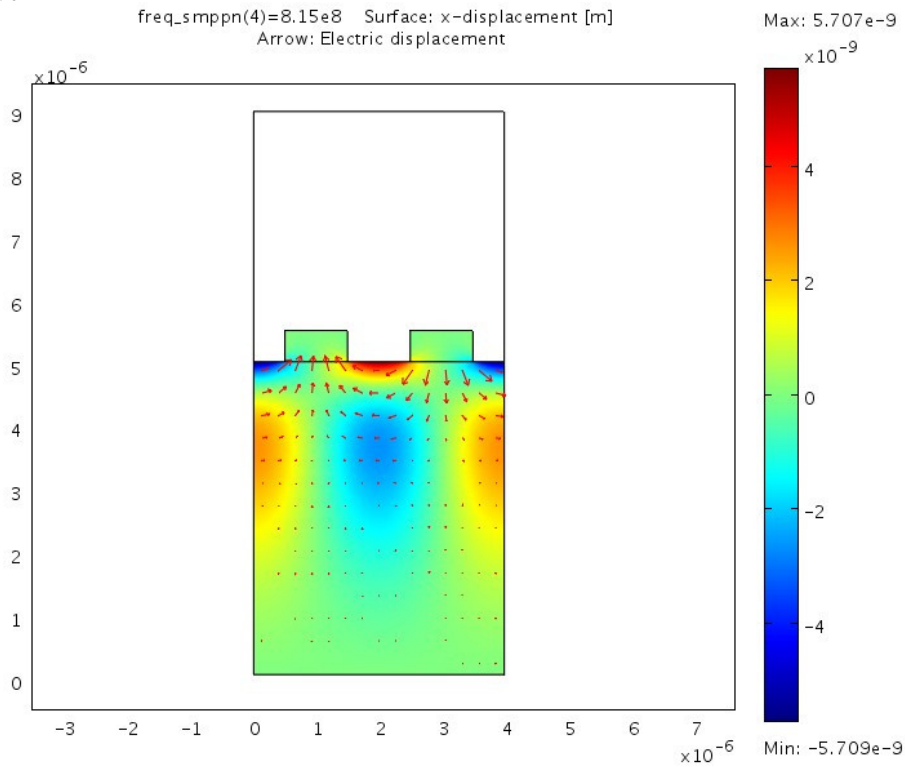
6 A guitar string

The equation of vibrating strings is

$$T\frac{\partial^2 u}{\partial x^2} = \mu\frac{\partial^2 u}{\partial t^2} \tag{1}$$

with μ the mass per unit length and T the tension (a force, expressed in Newton). This equation is actually a 1D wave equation. The wave velocity is $v = \sqrt{T/\mu}$ (check the physical units!).

We consider a string with length $L = 1$ m, and square cross section with side length 1 mm. We consider $\mu = 3$ g/m and $T = 100$ N. The string is clamped at both ends.

1. Model the problem in 3D, PDE mode, Wave equation. The long length is along x. Find the eigenvalues (with solver "eigenvalue"), and derive the first 8 eigenfrequencies.

2. Do eigenfrequencies follow the harmonic progression $f_n = n\frac{v}{2L}$?

3. Produce an animation for one period in time of the third mode of vibration (with u in color scale and deforming the mesh along u in direction z).

4. Is it surprising to be able to model a 1D wave equation using a 3D mesh?