Master PICS, TD #4: Piezoelectricity, resonances, opto-acoustics, acousto-optics

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January 5, 2021

1 Opto-acoustics in a microwire

The mixing of two frequency detuned optical waves in a solid medium offers a means of generating coherent hypersound in the bulk. Such an electrostriction process relies on the photoelastic effect, whereby the square of the optical field induces a mechanical stress in the medium. When the stress distribution is phase matched with a particular elastic wave in the medium, i.e. when the interference wavelength and the detuning frequency satisfy the relevant dispersion relation, optoacoustic generation is in principle at a maximum.

We consider a homogeneous sikica microwire, with diameter $d = 1$ µm. The magnetic fields of the two optical beams are linearly polarized and have a transverse Gaussian dependence, according to

$$\mathbf{H}_i(x, y, z; t) = H_0 \mathbf{h}_i \exp(-(x^2 + y^2)/w^2) \exp(j(\omega_i t - k_i z)) \tag{1}$$

with $i = 1, 2$, H_0 the magnetic field magnitude, $\mathbf{h}_i$ a transverse unit vector, and w the beam waist. ω_i and k_i are the angular frequency and wavevector, with $|k_i| = n\omega_i/c$ and c the vacuum velocity of light. The power transported by the Gaussian beam is in the paraxial approximation

$$P_0 = \frac{\pi w^2}{4n\varepsilon_0 c} H_0^2. \tag{2}$$

The two optical beams are counterpropagating and have a relatively small adjustable detuning $\omega = \omega_1 - \omega_2$. Hence the difference of their wavenumbers $q = k_1 - k_2 \approx 2k_1$. The square of the optical field thus contains a term proportional to $\exp(j(\omega t - qz))$ that can be phase-matched with an elastic wave providing $q \approx \omega/V$ with V the phase velocity of the elastic wave. The relevant term in the electrostriction stress tensor is $T^{es}_{kl} = -\frac{1}{2}\varepsilon_0 p_{ijkl}\ D^{(1)}_i D^{(2)*}_j$ with $\mathbf{D}$ the electric displacement vector and p_{ijkl} the photoelastic tensor. The electric displacement vector is easily obtained for the magnetic field vector by $\frac{\partial \mathbf{D}}{\partial t} = \nabla \times \mathbf{H}$.

Given the particular form of the driving electrostriction stress, the displacement of the forced elastic wave can be assumed to be of the form

$$u_i(x, y, z; t) = \hat{u}_i(x, y) \exp(j(\omega t - qz)). \tag{3}$$

This equation looks similar to a guided mode but actually describes a different situation: the driving stress oscillates in time and space along a preferred direction and the forced solution follows those oscillations while adapting its shape in the transverse plane to satisfy the appropriate boundary conditions.

We obtain the displacements $\hat{u}_i(x, y)$ by solving the elastodynamic equation using finite element analysis. The weak form of the equation is

$$-\omega^2 \int_\Omega \rho \hat{v}_i^* \hat{u}_i + \int_\Omega S(\hat{v})^*_I c_{IJ} S(\hat{u})_J = \int_\Omega S(\hat{v})^*_I T^{\mathrm{es}}_J, \tag{4}$$

with c_{ijkl} the elastic tensor and ρ the mass density. Ω is the domain on which $\hat{u}_i(x,y)$ is defined, $\mathbf{v}$ is a set if test functions satisfying the decomposition of (3). The strains are defined as

$$
\begin{aligned}
S_1(\hat{\mathbf{u}}) &= \frac{\partial \hat{u}_1}{\partial x_1}, && (5)\\
S_2(\hat{\mathbf{u}}) &= \frac{\partial \hat{u}_2}{\partial x_2}, && (6)\\
S_3(\hat{\mathbf{u}}) &= -jq\hat{u}_3, && (7)\\
S_4(\hat{\mathbf{u}}) &= \frac{\partial \hat{u}_3}{\partial x_2} - jq\hat{u}_2, && (8)\\
S_5(\hat{\mathbf{u}}) &= \frac{\partial \hat{u}_3}{\partial x_1} - jq\hat{u}_1, && (9)\\
S_6(\hat{\mathbf{u}}) &= \frac{\partial \hat{u}_1}{\partial x_2} + \frac{\partial \hat{u}_2}{\partial x_1}. && (10)
\end{aligned}
$$

1. Obtain the elastic modes of the microwire, for a fixed $q = 2n\omega/c = 2n/\lambda$, with $\lambda = 1.55$ µm. For that purpose, study and understand script `microwire_modes.edp`. Do we have many elastic modes in the dispersion relation?

2. Obtain the electrostriction response, subject to the force above, for frequencies around the first mode. For that purpose, study and modify script `microwire.edp` as you like. What happens exactly at the resonance frequency, i.e. at the frequency of the mode? [indication: refine the frequency step to increase the resolution]

3. Add a phenomenological loss term. Viscoelastic losses are included by adding an imaginary part to the elastic tensor c_{ijkl}, proportional to the viscosity tensor η_{ijkl} and to angular frequency Ω

$$
c_{ijkl} \leftarrow c_{ijkl} + \imath\Omega\eta_{ijkl}. \tag{11}
$$

   What changes?

2 Piezoelectricity

We consider piezoelectricity as an example of coupling included inside the partial differential equations. We consider a thick plate of lithium niobate (thickness 10 µm, side 100 µm), placed between two thin metal sheets. We are considering the generation of charges (a current if the metal sheets are connected by an electrical circuit) at the top and bottom surfaces under a fixed harmonic electric potential. The source is represented by the boundary conditions.

1. Study and understand script `piezo.edp`. How are the charges computed? What are G and B that are computed?

3 Acousto-optics in a nanostructure

Example of coupling occuring because of a deformation of the structure. Illustration with a presentation taken from a publication in Optica in 2017.