Master PICS,
TD #3: Vector waves, elasticity, PML

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1 Static deformation of a silicon beam

Consider a silicon beam clamped on the left side. The beam is 10 microns long and 500 nm thick. We neglect in a first approximation the third dimension and we want to analyze the bending of the beam under the action of gravity: given its small mass and its rigidity, by how much does it bend naturally?

1. Study and understand script `static.edp`. What equation is implemented? What are the material constants and how are then defined? What are the boundary conditions?

2. Run the script and observe the output. How is the deformed mesh obtained?

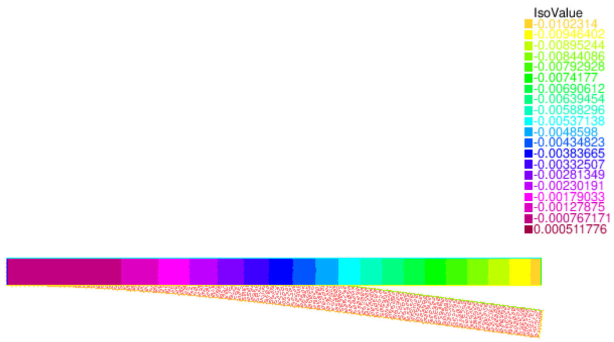

Figure 1: Static deformation of a silicon beam under gravity.

2 Modes of deformation of a silicon beam

We consider again the same silicon beam. We now want to obtain the possible modes of deformation and their frequencies.

1. Study and understand script `modes.edp`. Which equation is now solved and how is it different from the previous one? Why do we have modes deforming in-plane and others only along the third dimension?

2. Run the script and observe the different modal shapes. Play with the scaling factor to animate the deformed mesh.

3. Change the boundary condition to: **all free**. Run again and discuss the changes?

4. Change the boundary condition to: **left and right clamped**. Run again and discuss the changes?

3 Phononic crystal slab in silicon

We consider a thin membrane in silicon. When processed to form a periodic array of holes (or inclusions), a phononic crystal slab is formed (see Figure 2).

1. Study and understand script `bs_slab.edp` and the various files it includes. What is different from the band structure scripts given in TD1?

2. Run the script and observe how the mesh is defined (it can be rotated in the 3D view).
   Note: This is a fully featured script used to generate band structures in publications. *It is complicated.*

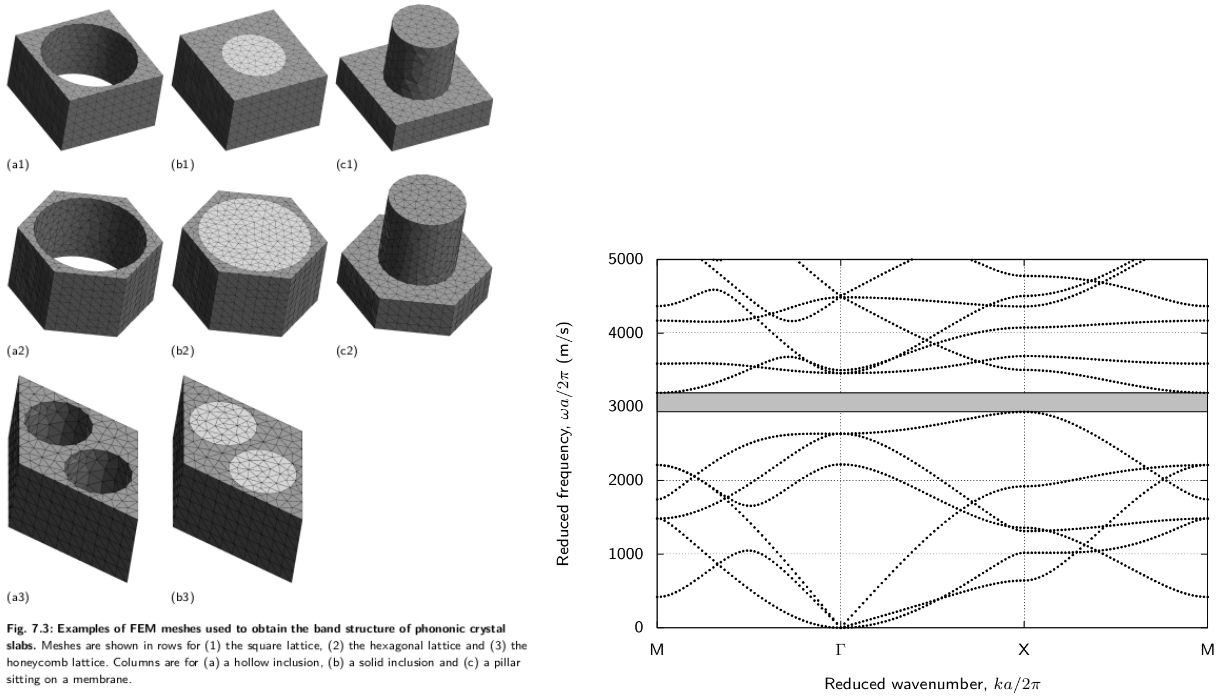

Figure 2: (a) Examples of FEM meshes for the periodic unit-cell of a phononic crystal slab. (b) Band structure of a square lattice phononic crystal slab of circular holes in silicon ($d/a = 0.9$).

4 PML for acoustic waves

We come back to the finite 2D phononic crystal considered in TD1 (question 2). We will replace the radiation boundary condition by a perfectly matched layer surrounding the computation domain. The case of the circularly symmetric PML is considered, because it is well suited for radiation problems.

1. Compare script `src2d_source.edp` with script `acoustic_source_radiation.edp` of TD1. What is different and what is not?

2. Consider script `phononic_crystal.edp` that was given as a solution to TD1 (a crystal of 7×4 steel rods). Can you write a script that replaces the radiation boundary condition with the same PML as above?

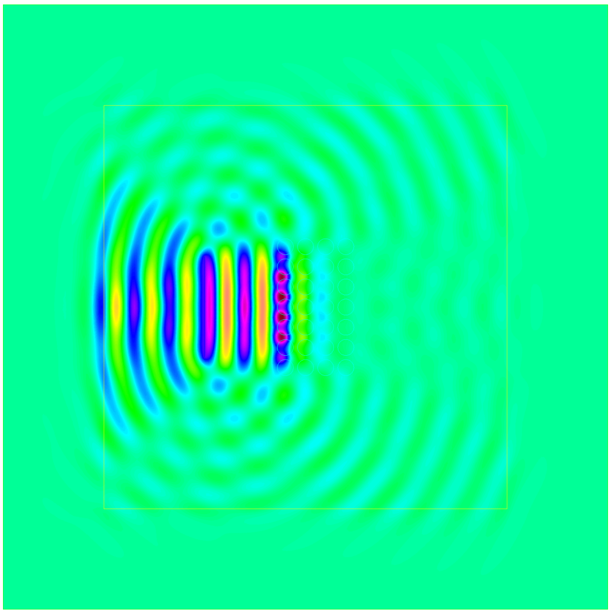

Figure 3: PML for an acoustic wave incident on a small finite phononic crystal of steel rods in water.