Master PICS,
TD #2: Optical and acoustic waveguides

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1 Acoustic waveguide

Consider a hollow tube filled with air, like an organ pipe for instance. We assume the tube is very rigid, so that acoustic waves in air hit a hard wall that will not move, reflecting them completely. We want to obtain the dispersion of the acoustic waves guided inside the tube and to look at the modal shapes.

1. Study and understand script `acoustic_wg.edp`. What equation is implemented? What are the material constants defining the air? What are the boundary conditions?
2. Run the script and plot the dispersion relation. An example `wg.plt` gnuplot script is given to plot Figure 1. Comment the different modes that are obtained; are some of them degenerate? How does the dispersion depend on the diameter of the tube?
3. Modify the ff++ script in order to plot the modal shapes for k_z equal one third of k_{max}. Can you comment on the symmetries of the modes with respect to the symmetry of the waveguide?
4. (If you have ample time only) Modify script `acoustic_wg.edp` in order to consider a different, arbitrary, cross-section of the waveguide, for instance a triangle or a rectangle. Do all modes change? Why?

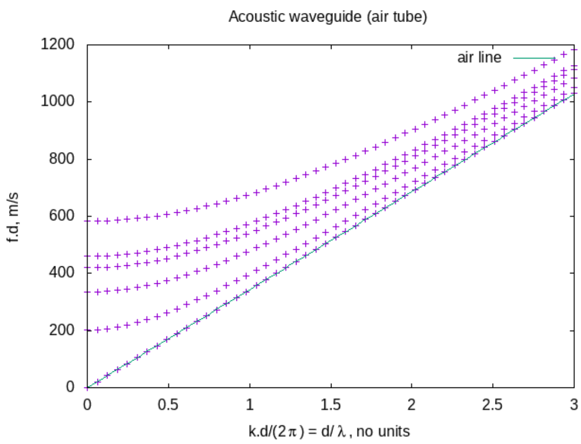

Figure 1: Dispersion relation of an acoustic waveguide.

2 Complex dispersion of the acoustic waveguide

We consider again the same waveguide as before. We now want to obtain the dispersion relation in the form $k_z(\omega)$, i.e. we fix the frequency and solve for the possible spatial wavenumbers.

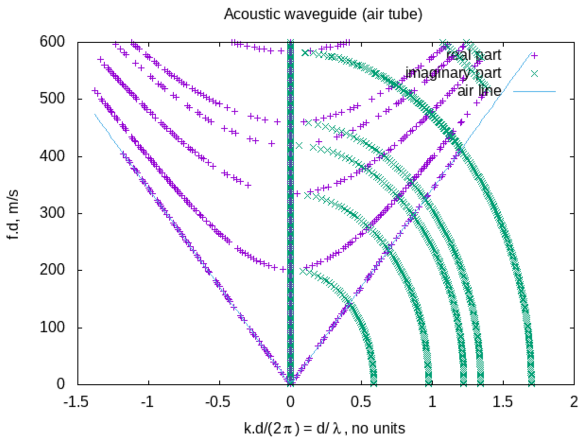

Figure 2: Complex dispersion relation of an acoustic waveguide.

1. Study and understand script `acoustic_wg_complex.edp`. Which equation is now solved and how is it different from the previous one?
2. Run the script and plot the complex dispersion relation. An example `wg_complex.plt` gnuplot script is given to plot Figure 2. Do you understand the physical meaning of the purely imaginary branches that now appear? (Note: they are evanescent guided waves.)

3 Silica microwire

Maxwell's equations in dielectric media lead to the following vector wave equation for the magnetic field vector $\boldsymbol{H}$

$$\nabla \times \left(\frac{1}{\epsilon} \nabla \times \boldsymbol{H} \right) - \frac{\omega^2}{c^2} \boldsymbol{H} = \boldsymbol{f}, \tag{1}$$

with ϵ the relative dielectric constant and c the speed of light in a vacuum. Because of invariance of the structure along axis x_3, solutions can be written as $\bar{\boldsymbol{H}}(x_1, x_2) \exp(\imath(\omega t - k x_3))$. Hence the unknown becomes the modal shape $\bar{\boldsymbol{H}}(x_1, x_2)$ defined in two-dimensional transverse space. As a result, it is enough the represent the solution on a two-dimensional mesh.

A weak form of the guided-wave optical equation is

$$\int_\Omega \frac{1}{\epsilon} \left(\mathrm{rot}(\bar{\boldsymbol{H}}')\mathrm{rot}(\bar{\boldsymbol{H}}) + \mathrm{div}(\bar{\boldsymbol{H}}')\mathrm{div}(\bar{\boldsymbol{H}}) + k^2 \bar{\boldsymbol{H}}' \cdot \bar{\boldsymbol{H}} \right) + \int_{\delta\Omega} \bar{H}'_n \mathrm{div}(\bar{\boldsymbol{H}}) \left[\frac{1}{\epsilon} \right] - \frac{\omega^2}{c^2} \int_\Omega \bar{\boldsymbol{H}}' \cdot \bar{\boldsymbol{H}} = \int_\Omega \bar{\boldsymbol{H}}' \cdot \bar{\boldsymbol{f}}. \tag{2}$$

This expression is obtained by keeping as unknowns only the first two components of $\boldsymbol{H}$, i.e. $\boldsymbol{H} = (H_1, H_2)$, since the third component is set by the auxiliary Maxwell equation $\nabla \cdot \boldsymbol{H} = 0$. Here we use the transverse divergence $\mathrm{div}(\bar{\boldsymbol{H}}) = \bar{H}_{1,1} + \bar{H}_{2,2}$ and transverse rotational $\mathrm{rot}(\bar{\boldsymbol{H}}) = \bar{H}_{2,1} - \bar{H}_{1,2}$, $\bar{H}'_n$ is the normal component of $\bar{\boldsymbol{H}}'$ at the boundary $\delta\Omega$, and $\left[\frac{1}{\epsilon}\right]$ denotes the jump of the permittivity. Note that the boundary integral appears because of the non continuity of the electric field at the interface between different dielectric media.

1. We consider a silica microwire standing in air or in a vacuum, with a diameter of 1 µm. Study and understand script `fiber_silica.edp`. What is really different compared to the acoustic case, except for the equations?
2. Run the script and plot the dispersion relation. An example `fiber.plt` gnuplot script is given to plot Figure 3. What are the light and silica lines, and what are they useful for?
3. Repeat the same computation but for a silicon microwire. What changes?
4. Obtain the modal shapes for some value of k_3 you can choose.

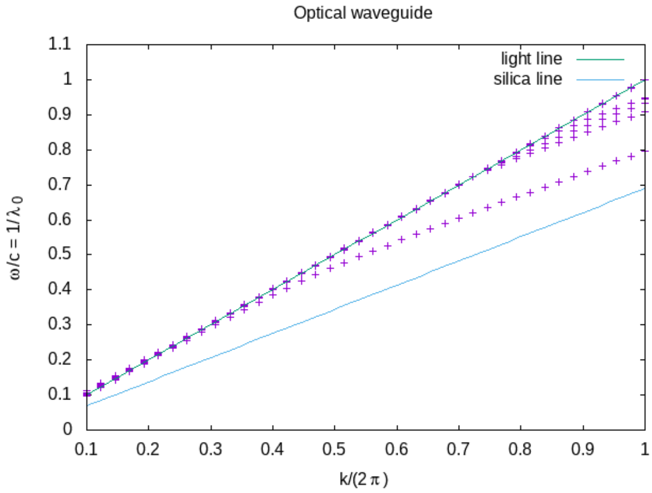

Figure 3: Dispersion relation of a silica microwire.

4 Nanoscale planar waveguide [problem]

We want to obtain the dispersion of the nanoscale planar waveguide whose mesh is depicted in Figure 4. A thin wire of silicon (width 800 nm, height 350 nm) sits on a silica substrate, and air is on top. Obtain the dispersion relation, inspiring from the previous section.

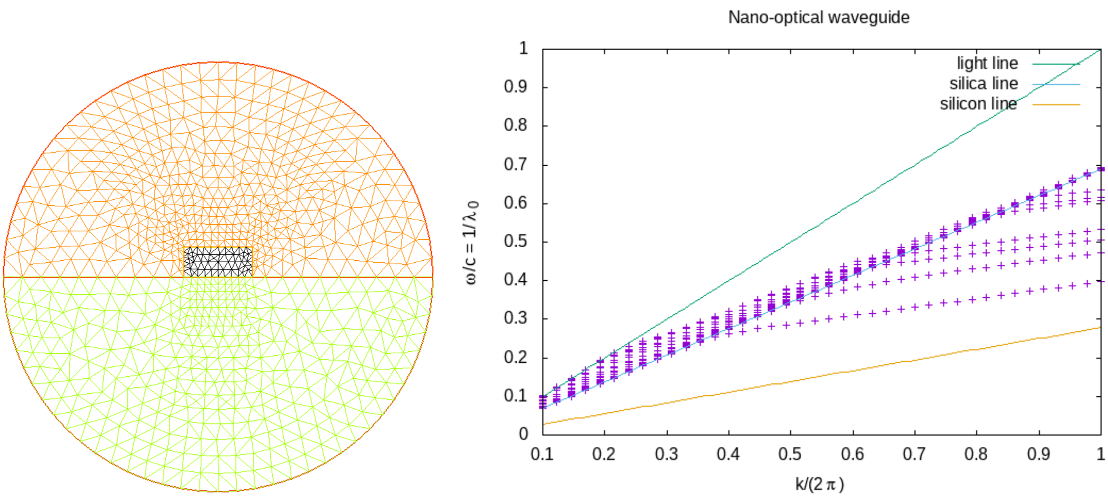

Figure 4: Dispersion relation of a nanoscale planar waveguide.