Guided elastic waves

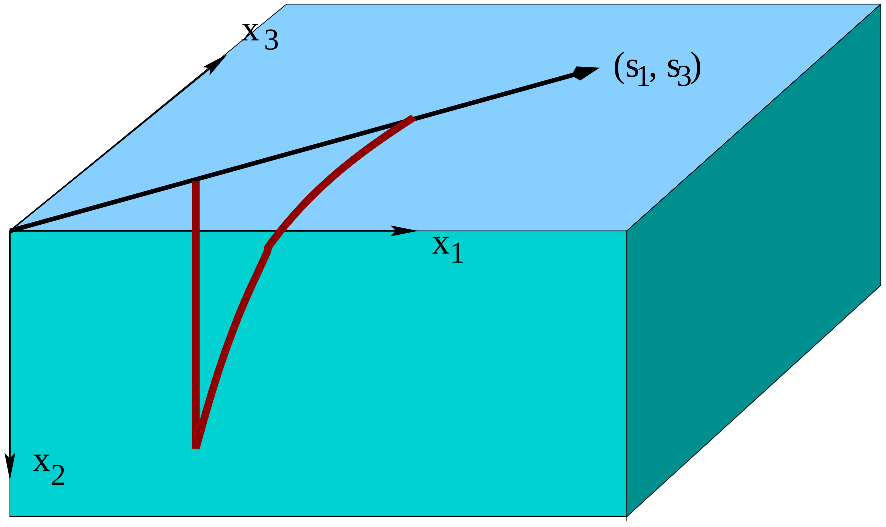
by Vincent Laude

Institut FEMTO-ST, MN2S department
group « Phononics & Microscopy »
15B avenue des Montboucons F-25030 Besançon

Email: vincent.laude@femto-st.fr
Web: http://members.femto-st.fr/vincent-laude/

1 Surface waves

A surface wave is a solution of the wave equation satisfying surface boundary conditions on top of a semi-infinite substrate in a vacuum. The surface wave is guided if its amplitude decays in the depth.



We assume the surface slowness is imposed (i.e. the wavevector), or s_1 and s_3 are given. Only the reflected partial waves can contribute to the surface wave (that is, 3 partial waves for an elastic solid and 4 for a piezoelectric solid).

Elastic boundary condition: for a free surface, $T_{i2} = 0$.

Electric boundary condition (piezoelectric case): for a free surface, zero charge on the surface; for a short-circuited metalized surface (e.g., if an infinitely thin film of metal is present), $\phi = 0$.

1.1 Selection rule for partial waves (reflected)

Rule : Slowness curves are explored in the sagittal plane (normal to the surface and containing the direction of propagation).

- A propagating partial wave (s_2 real) is selected if the Poynting vector is oriented toward the inside, or $P_2 > 0$.
- An inhomogeneous partial waves is selected if $\mathrm{Im}(s_2) < 0$, so that its amplitude decays inside the substrate.

Reflected partial waves are renumbered from 1 to 3 (elastic solid) or 1 to 4 (piezoelectric solid).

Form of the solution: superposition of reflected partial waves

$$\boldsymbol{h}(t, \boldsymbol{x}) = \sum_{r=1}^{3\mathrm{or}4} a_r \boldsymbol{h}_r \exp\left(-\mathrm{i}\omega s_{2r} x_2) \right) \exp(\mathrm{i}\omega (t - s_1 x_1 - s_3 x_3) \tag{1}$$

with $h_l = (u_l, \tau_{l2})$ the vector of generalized displacements and stresses, s_{2r} and $\boldsymbol{h}_r$ the eigenvalues and eigenvectors characterizing the partial waves, and a_r the amplitudes of partial waves (unknown at first).

1.2 Case of elastic solids

Expression of the boundary condition determinant $T_{i2} = 0$ for $x_2 = 0$?
There are 3 boundary conditions for 3 unknown amplitudes:

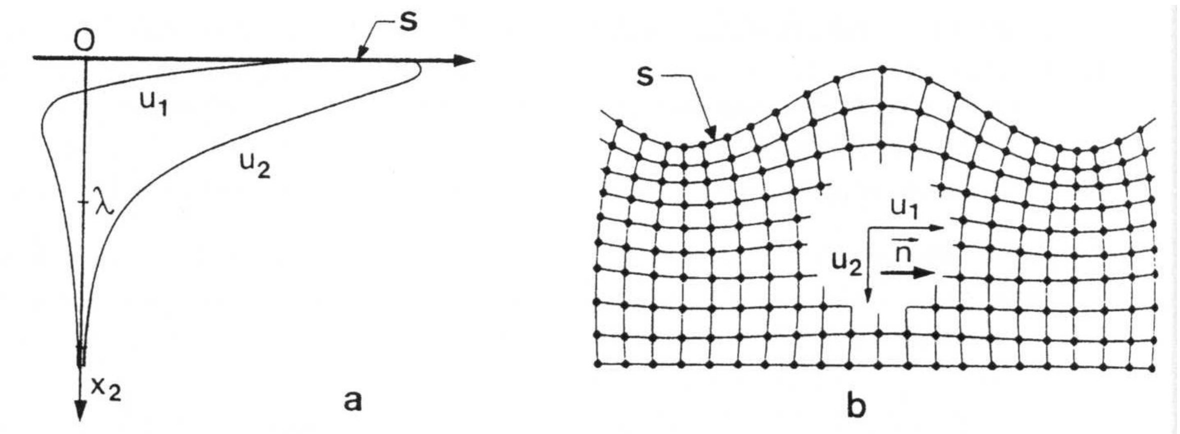
$$\tau_{i2} = \sum_{r=1}^{3} a_r \tau_{i2}^{(r)} = 0, \; i = 1, 2, 3 \tag{2}$$

This equation has a solution only if the boundary condition determinant is zero:

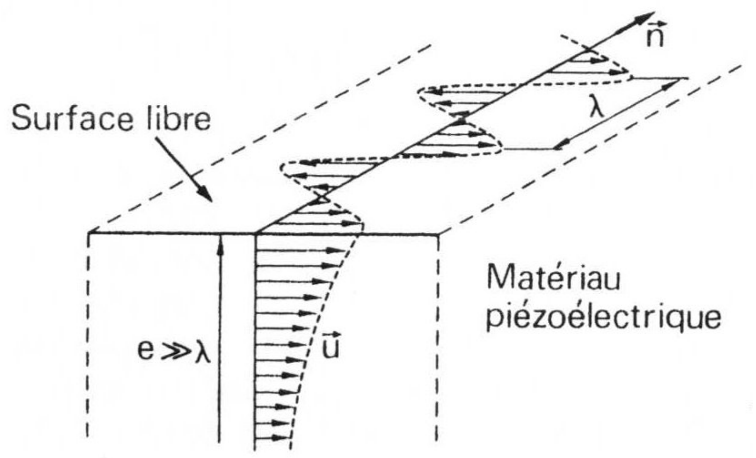
$$\Delta = |\tau_{i2}^{(r)}| = \begin{vmatrix} \tau_{12}^{(1)} & \tau_{12}^{(2)} & \tau_{12}^{(3)} \\ \tau_{22}^{(1)} & \tau_{22}^{(2)} & \tau_{22}^{(3)} \\ \tau_{32}^{(1)} & \tau_{32}^{(2)} & \tau_{32}^{(3)} \end{vmatrix} = 0 \tag{3}$$

The determinant is a function of s_1 and s_3 (of the surface slownesses). The equation defines a slowness curve for surface guided waves. The surface wave is perfectly guided if all partial modes are inhomogeneous: the velocity of the surface wave is then less than the velocities of all bulk waves.
Most of the time, we consider Rayleigh waves, whose polarization (i.e. the displacements) is purely sagittal. Bleustein-Gulyaev waves (that exist only at the surface of piezoelectric solids) are purely transverse waves.

1.3 Rayleigh and Bleustein-Gulyaev waves

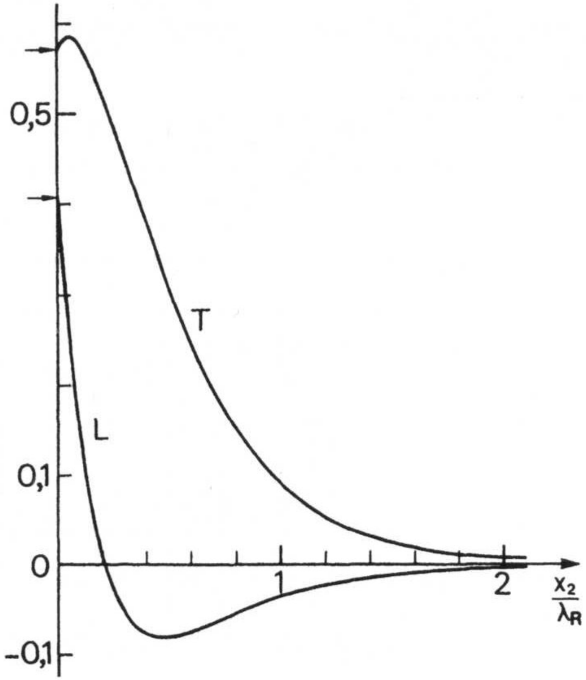


Rayleigh wave (R)

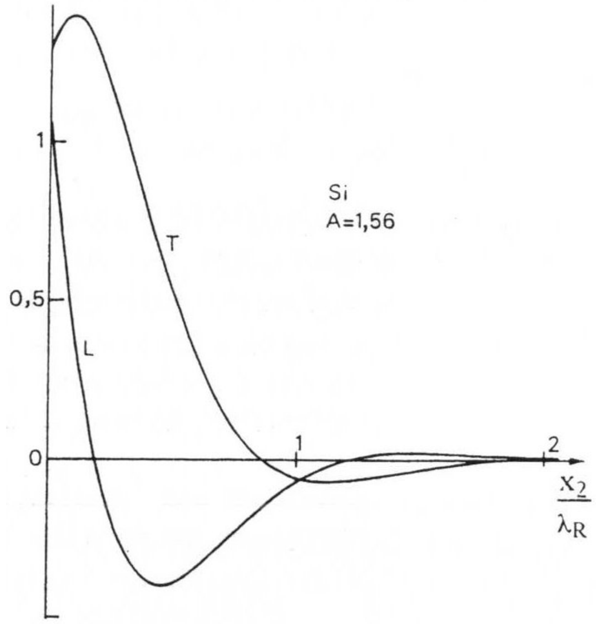


Bleustein-Gulyaev wave (BG)

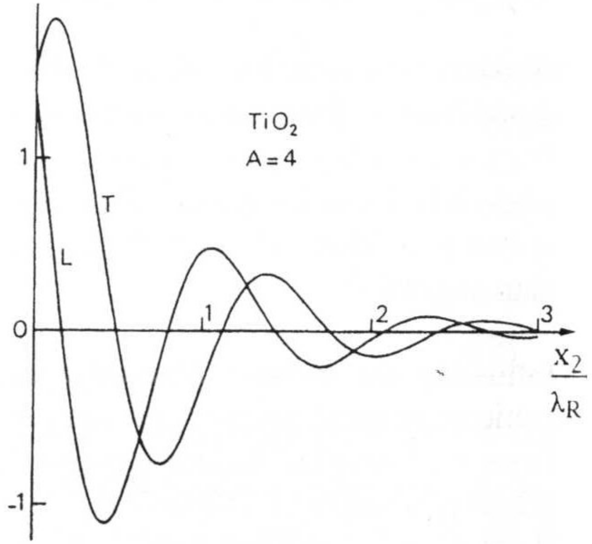
1.4 Examples of Rayleigh waves



Silica
(isotropic)

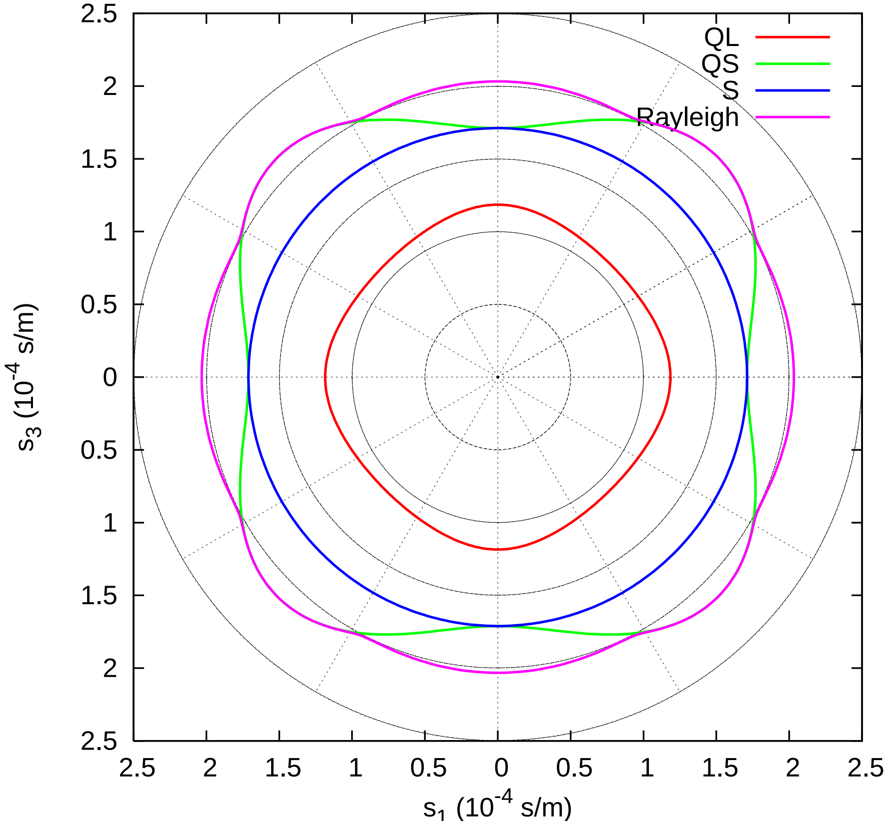


Silicium
(plane (010), prop. [100])

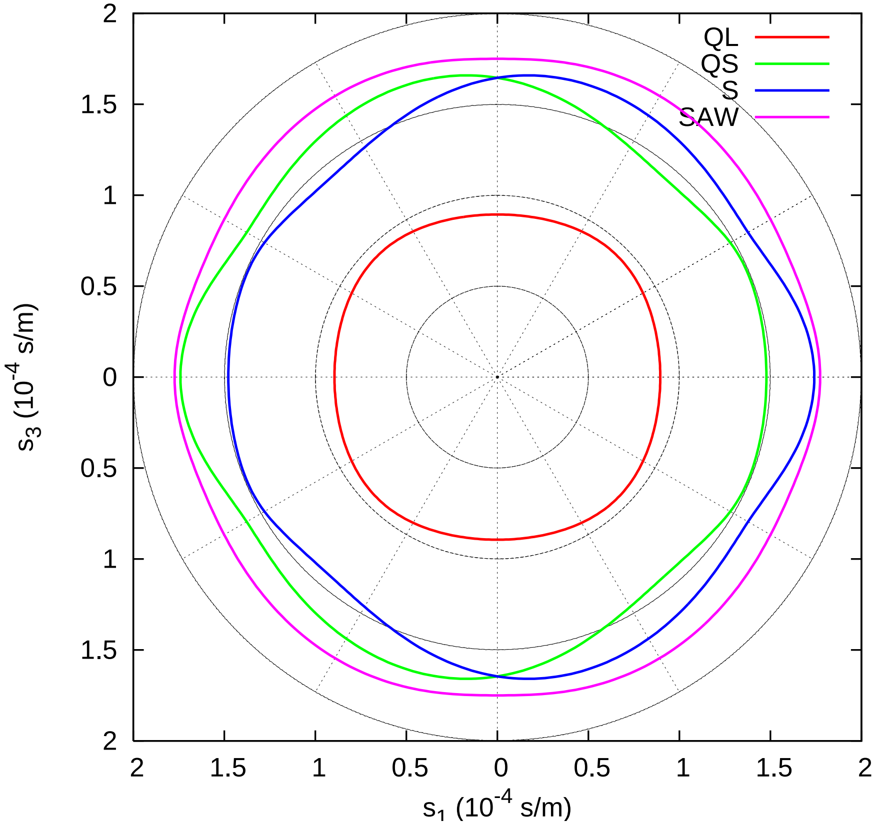


TiO_2
(plane (010), prop. [100])

1.5 Slowness curves for silicon and sapphire



Silicon, Z cut or plane (001)
Rayleigh surface wave



Sapphire, Y cut or plane (010)
Generally polarized surface wave

1.6 Piezoelectric case

Free surface There are now 4 reflected partial waves. There are no surface charges, or $\Delta D_2 = 0$ (the electric displacement is continuous).

On the vacuum side, the electric potential satisfies Poisson's equation $s_i\, s_i\, \phi = 0$, and thus (the sign is selected so that the solution is evanescent in a vacuum) $s_2 = +\mathrm{i}\sqrt{s_1^2 + s_3^2}$. The normal component of the electric displacement on surface $x_2 = 0$ is then $D_2 = -s_2\, \varepsilon_0\, \phi = -\mathrm{i}\sqrt{s_1^2 + s_3^2}\, \varepsilon_0\, \phi$.

In addition to zero surface traction (2), we have on surface $x_2 = 0$

$$D_2 + \mathrm{i}\, s\, \varepsilon_0\, \phi = 0 \text{ with } s = \sqrt{s_1^2 + s_3^2}$$

The boundary condition determinant becomes

$$\Delta_F = \begin{vmatrix} \tau_{12}^{(1)} & \tau_{12}^{(2)} & \tau_{12}^{(3)} & \tau_{12}^{(4)} \\ \tau_{22}^{(1)} & \tau_{22}^{(2)} & \tau_{22}^{(3)} & \tau_{22}^{(4)} \\ \tau_{32}^{(1)} & \tau_{32}^{(2)} & \tau_{32}^{(3)} & \tau_{32}^{(4)} \\ D_2^{(1)} + \mathrm{i}\, s\, \varepsilon_0\, \phi^{(1)} & D_2^{(2)} + \mathrm{i}\, s\, \varepsilon_0\, \phi^{(2)} & D_2^{(3)} + \mathrm{i}\, s\, \varepsilon_0\, \phi^{(3)} & D_2^{(4)} + \mathrm{i}\, s\, \varepsilon_0\, \phi^{(4)} \end{vmatrix} \qquad (4)$$

$\Delta_F = 0$ is the condition of existence of the surface wave.

Short-circuited surface In addition to zero surface traction (2), the electric potential is zero at $x_2 = 0$:

$$\phi = \sum_{r=1}^{4} a_r \phi^{(r)} = 0 \tag{5}$$

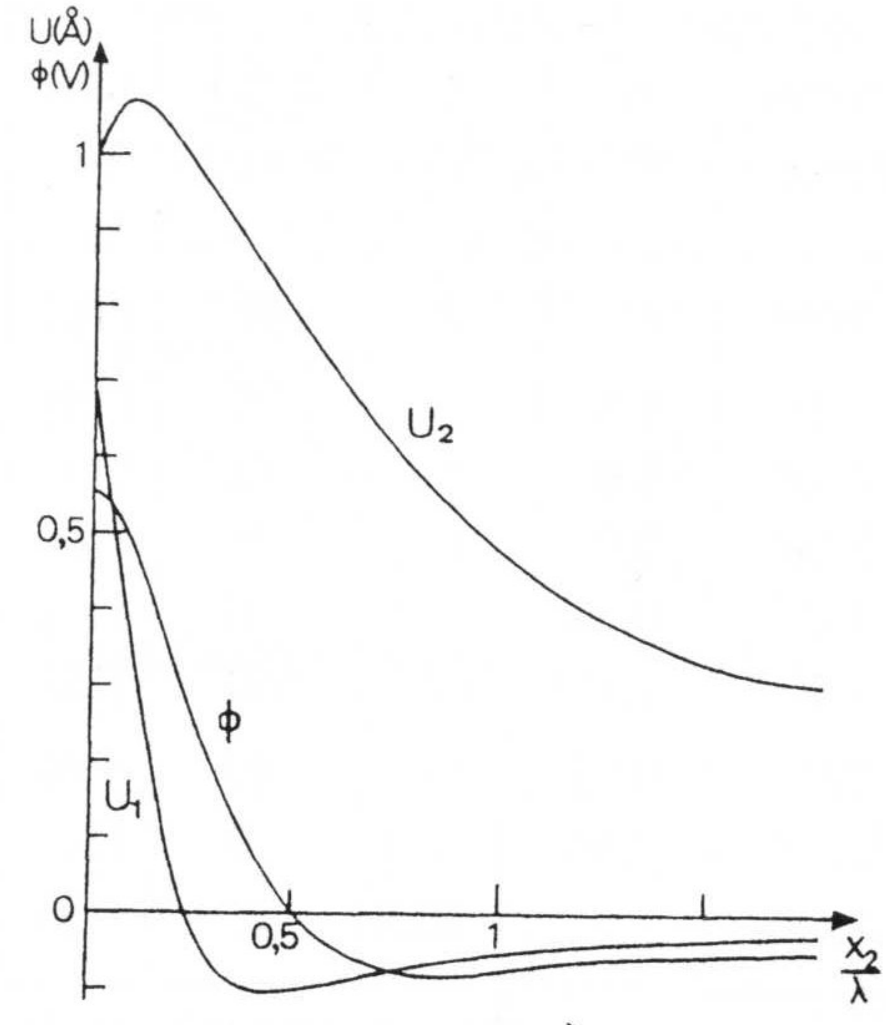
The boundary condition determinant becomes

$$\Delta_M = \begin{vmatrix} \tau_{12}^{(1)} & \tau_{12}^{(2)} & \tau_{12}^{(3)} & \tau_{12}^{(4)} \\ \tau_{22}^{(1)} & \tau_{22}^{(2)} & \tau_{22}^{(3)} & \tau_{22}^{(4)} \\ \tau_{32}^{(1)} & \tau_{32}^{(2)} & \tau_{32}^{(3)} & \tau_{32}^{(4)} \\ \phi^{(1)} & \phi^{(2)} & \phi^{(3)} & \phi^{(4)} \end{vmatrix} = 0 \tag{6}$$

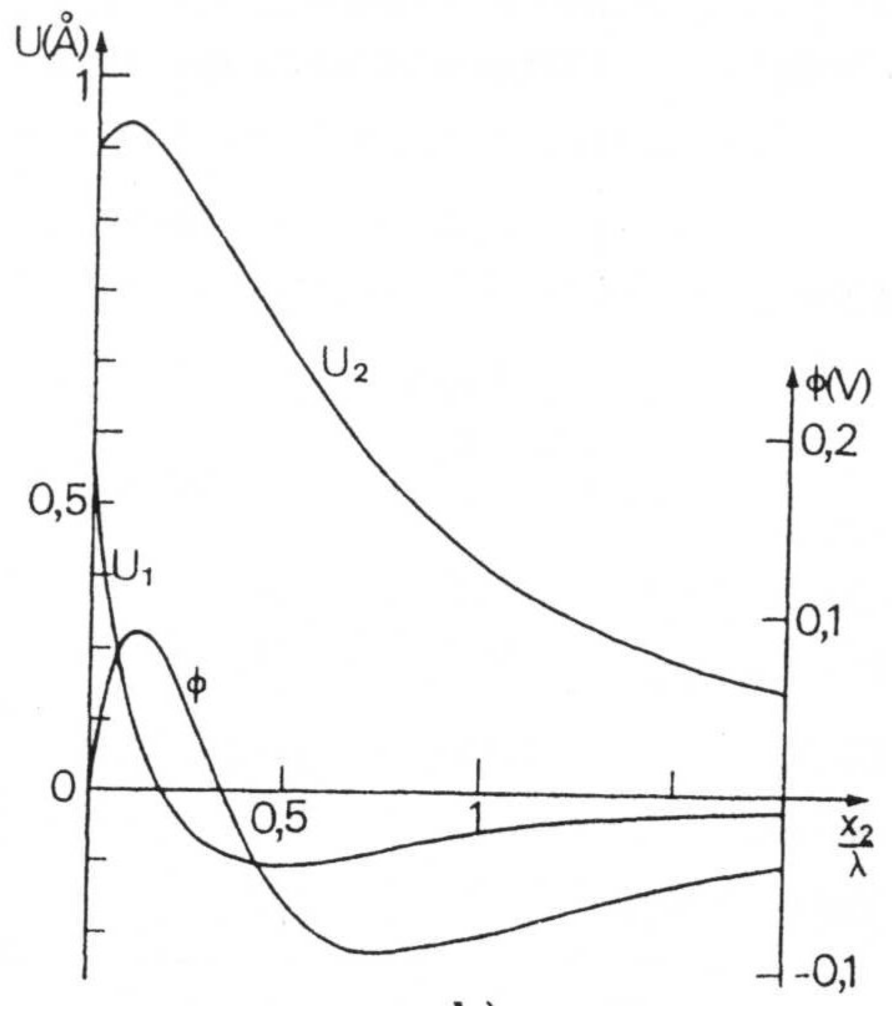
Coupling The two boundary condition determinants Δ_F et Δ_M each give a different slowness curve for the surface wave. The slowness on the metal surface is always larger than the slowness on the free surface (the velocity is smaller). The electromechanical coupling is approximately given by

$$K^2 = \frac{v_F - v_M}{v_F + v_M} = 2\frac{\Delta v}{v} = 2\frac{\Delta s}{s}$$

1.7 Example: lithium niobate (Y cut, propagation Z)

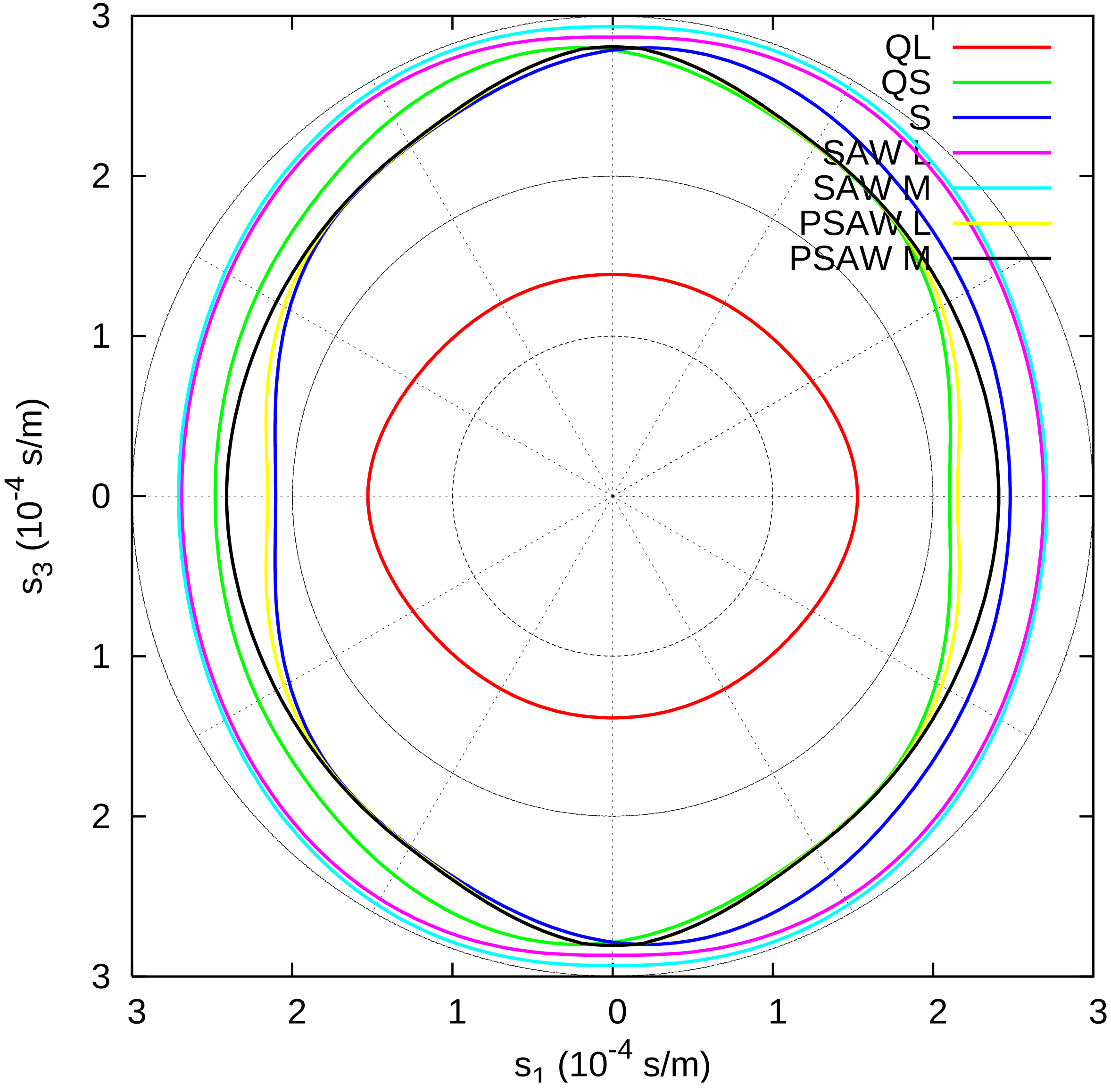


free surface



short-circuit metal surface

1.8 Slowness curves for lithium niobate, Y cut



1.9 Green's function and effective surface permittivity

Spectral Green's function Whatever the surface boundary conditions, the number of relations in (1) is double the number of reflected partial waves. We can eliminate the amplitudes to obtain a matrix relation at the surface

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ \phi \end{pmatrix} = G \begin{pmatrix} \tau_{12} \\ \tau_{22} \\ \tau_{32} \\ \frac{D_2}{-\mathrm{i}\omega} \end{pmatrix} \text{with } G = \left(\bar{u}_i^{(r)} \right) \left(\bar{\tau}_{i2}^{(r)} \right)^{-1} \tag{7}$$

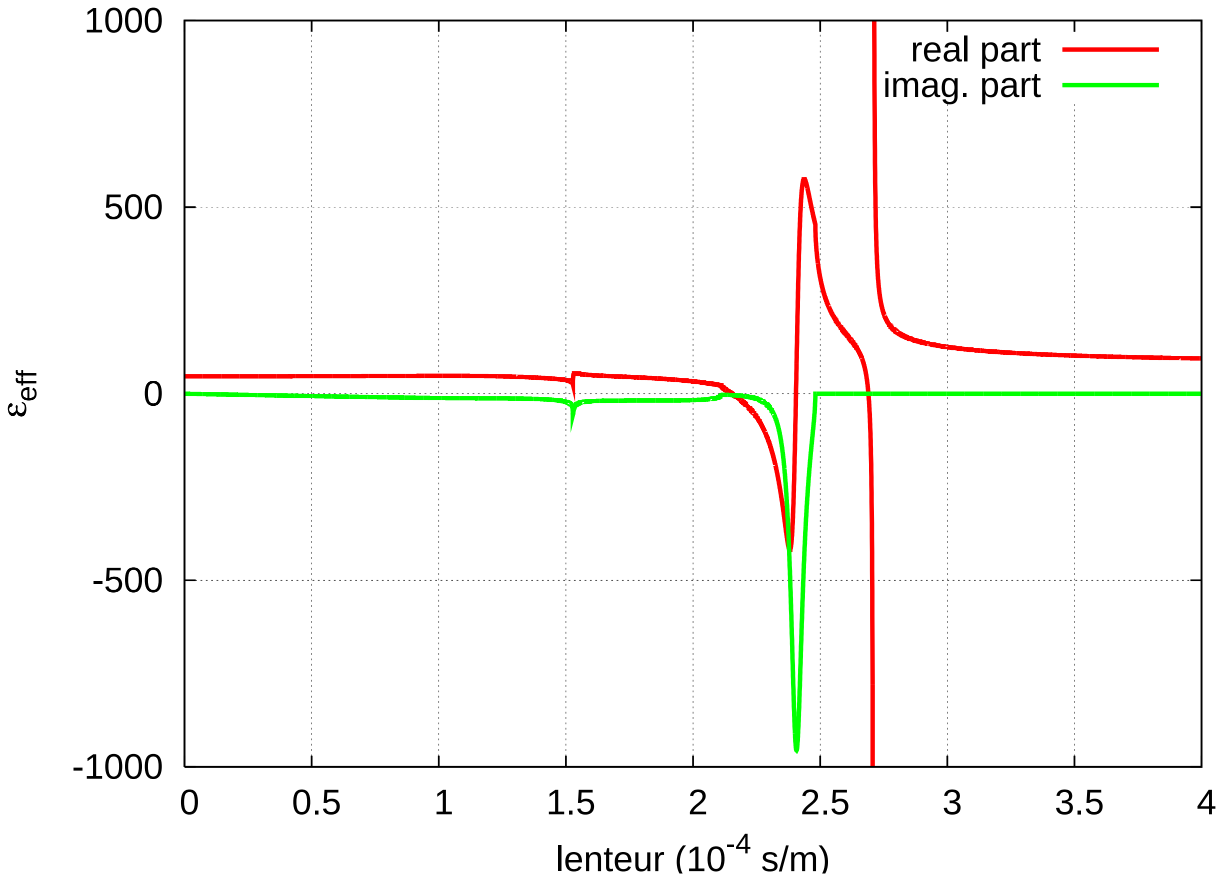
This relation allows us to consider an arbitrary surface excitation in numerical simulations.

Effective surface permittivity When the surface is mechanically free, stresses normal to the surface vanish and $\phi = G_{44} \frac{D_2}{-\mathrm{i}\omega}$. We then define

$$\varepsilon_{\text{eff}} = \frac{D_2}{E_{\text{longitudinal}}} = \frac{D_2}{-\omega s \phi} = \frac{\mathrm{i}}{s G_{44}} \tag{8}$$

The poles of this function give surface waves for a short-circuited surface; surface waves for a free surface are given by $\varepsilon_{\text{eff}} = -\varepsilon_0$.

1.10 Pseudo surface acoustic wave (PSAW)



←niobate Y, prop. X

When the surface boundary condition determinant is almost zero, but not strictly zero, a partially guided wave is obtained (i.e. a surface wave suffering radiation loss). In practice, those waves are very important since it is possible to balance limited loss with strong electromechanical coupling in a transducer.

1.11 Examples of surface acoustic waves used in practice

Crystal	Cut	Direction	Type	v_M (m/s)	α (dB/λ)	K^2 (%)	$\frac{\varepsilon_\infty}{\varepsilon_0}$
$LiNbO_3$	Y	Z	R	3390	0	4.5	46
$LiNbO_3$	Y+128	X	SAW	3870	0	5	56
$LiNbO_3$	Y+128	X	BG	4030	0	$2.5\ 10^{-4}$	56
$LiNbO_3$	Y+41	X	PSAW	4380	$2\ 10^{-2}$	16	63
$LiTaO_3$	Y	Z	R	3210	0	0.9	48
$LiTaO_3$	Y+36	X	PSAW	4110	$3\ 10^{-4}$	5.5	50
Quartz	Y+42	X	SAW	3158	0	0.11	6
ZnO	X	Y	BG	2823	0	0.5	10

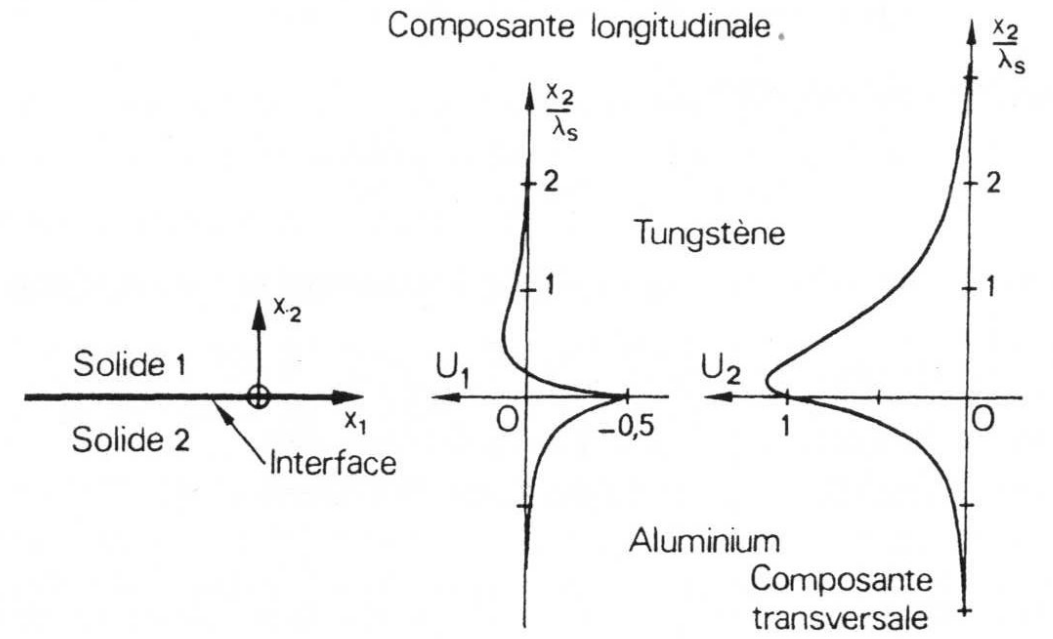
Table 1. Characteristics of surface waves on some crystal cuts.

Type of wave	Polarization
Rayleigh (R)	sagittal
Gereralized Rayleigh (SAW)	mainly sagittal
Bleustein-Gulyaev (BG)	a single transverse component
Pseudo SAW (PSAW)	lossy surface waves or leaky waves

Table 2. Classification of surface waves on solids.

2 Interface wave

An interface wave is a guided mode propagating at the interface between 2 materials. The interface wave is guided without loss if its displacement ampitude decreases exponentially on both sides of the interface. The interest of nterface waves is presently more theoretical than pratical, but they in principle allow one for achieving components whose active interface is protected (encapsulated).



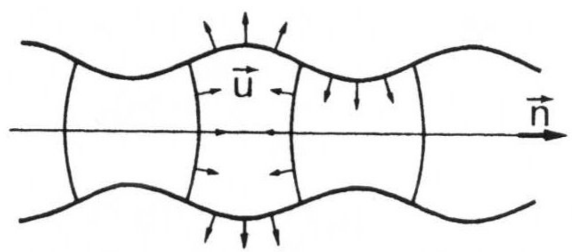
The concepts for surface waves can be extended to interface waves: boundary condition determinants, Green's function, effective interface permittivity, etc.

3 Plate modes

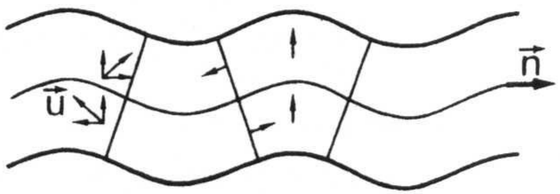
For a plate, surface boundary conditions must be considered simultaneously on both sides.

There exist bulk waves, surface waves if the thickness is large compared to the wavelength, and plate modes sensitive to both surfaces.

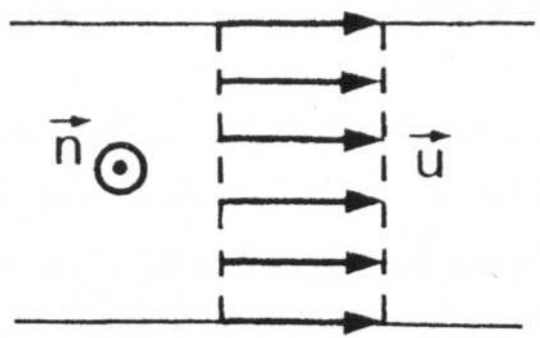
Lamb waves: plate waves with sagittal displacement in isotropic solids.



Symmetric mode



Antisymmetric mode



Purely transverse mode

4 Multilayer modes

In a multilayer (with thin or thick layers) there exist bulk waves, surface waves, interface waves, plate waves and waves of the multilayer strictly speaking. Computations still use the concepts of partial waves, of continuity relations between layers, and of boundary conditions.
In the case of a single isotropic layer on a semi-infinite isotropic substrate, there are purely transverse waves evanescent in the substrate that are called Love waves.