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Negative group velocities in metal-film optical waveguides

Pierre Tournois [a], Vincent Laude [b,*]

[a] Thomson-CSF, 173, boulevard Haussmann, F-75008 Paris, France
[b] Thomson-CSF Corporate Research Laboratory, Domaine de Corbeville, F-91404 Orsay Cedex, France

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Abstract

It is demonstrated that the group velocity of the frustrated transverse magnetic modes in an optical waveguide including a metal is negative when the power flow inside the metal is larger than the power flow inside the dielectric medium, while the phase velocity remains positive. The energy and the phase are then contrapropagating.

Keywords: Optical waveguides; Metal-film optical waveguides

Several types of optical waveguides [1] include a metal, e.g. the surface plasmon waveguide, the optical strip-line, the metal-film and the metal-clad optical waveguides (Fig. 1). For the sake of simplicity, we only consider in this paper the propagation of "frustrated" transverse-magnetic (F-TM) modes, for which the electrical and magnetic field vectors are given by hyperbolic functions. Furthermore, we consider only symmetrical waveguides (optical strip-line and metal-film waveguides).

The symmetrical waveguide depicted in Fig. 1 is a thin layer of thickness $2d$ and dielectric constant ϵ_2, sandwiched between identical semi-infinite media with dielectric constant ϵ_1. In such a waveguide, only a symmetrical (S) F-TM mode and an antisymmetrical (AS) F-TM mode can propagate. In the case of the symmetrical F-TM mode, the field amplitude H_y of the transverse magnetic field vector can be written:

$$H_y(x,z,t) = A\cosh(\alpha_2 x)\exp[\mathrm{i}(\omega t-\beta z)], \quad |x|\le d,$$
$$H_y(x,z,t) = B\exp(-\alpha_1|x|)\exp[\mathrm{i}(\omega t-\beta z)], \quad |x|\ge d, \tag{1}$$

with

$$\alpha_1^2=\beta^2-\epsilon_1\frac{\omega^2}{c^2}, \quad \alpha_2^2=\beta^2-\epsilon_2\frac{\omega^2}{c^2}, \tag{2}$$

where A and B are constants, ω is the optical angular frequency, c is the light velocity in vacuum and β is the propagation constant of the mode. Conversely, in the case of the antisymmetrical F-TM mode, the field amplitude H_y of the transverse magnetic field vector can be written:

$$H_y(x,z,t) = A\sinh(\alpha_2 x)\exp[\mathrm{i}(\omega t-\beta z)], \quad |x|\le d,$$
$$H_y(x,z,t) = B\exp(-\alpha_1|x|)\exp[\mathrm{i}(\omega t-\beta z)], \quad |x|\ge d. \tag{3}$$

The continuity of the field amplitudes H_y and $E_z = -(\mathrm{i}/\omega\epsilon)(\partial H_y/\partial x)$ at the interfaces $|x|=d$ leads to the dispersion relations:

$$\frac{\alpha_2}{\epsilon_2}\tanh(\alpha_2 d)+\frac{\alpha_1}{\epsilon_1}=0, \quad \text{(S) mode},$$
$$\frac{\alpha_2}{\epsilon_2}\coth(\alpha_2 d)+\frac{\alpha_1}{\epsilon_1}=0, \quad \text{(AS) mode}. \tag{4}$$

These relations do indeed have a solution only if ϵ_1 and ϵ_2 have opposite signs, i.e. if one of the media is a gas of electrons (plasma) or a metal whose plasma frequency is above the mode frequency.

In the following, we will consider only nonabsorbing metals (i.e. without ohmic losses), with refractive index

* Corresponding author. E-mail: laude@thomson-lcr.fr.

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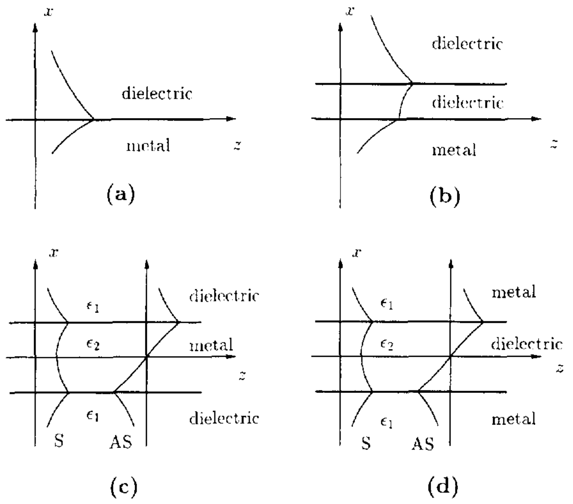

Fig. 1. Metal-film optical waveguides and associated F-TM modes: (a) the surface plasmon, (b) the metal-clad; (c) the metal-film; (d) the optical strip-line. S and AS stand respectively for symmetrical F-TM mode and antisymmetrical F-TM mode.

$n = -i\kappa$, or dielectric constant $\epsilon = -\kappa^2$. As an approximation to such metals, one can consider the use of silver (Ag), aluminum (Al), gold (Au) and copper (Cu) in both the visible and infrared regions [2].

As the thickness of the central layer becomes very large, the dispersion relations of Eq. (4) admit as a solution the plasmon surface wave whose phase velocity is given by

$$v_{\text{plasmon}} = (c/n)\sqrt{1 - n^2/\kappa^2}\,, \tag{5}$$

where n is the refractive index of the dielectric medium such that $\epsilon = n^2$. Obviously, this mode exists only when n is smaller than κ.

The phase velocity v_p and group velocity v_g are classically defined by

$$v_p = \omega/\beta, \quad v_g = \mathrm{d}\omega/\mathrm{d}\beta. \tag{6}$$

Under general assumptions [3], the group velocity is also given by the ratio of the net power flow of the Poynting vector to the electromagnetic energy density. The power

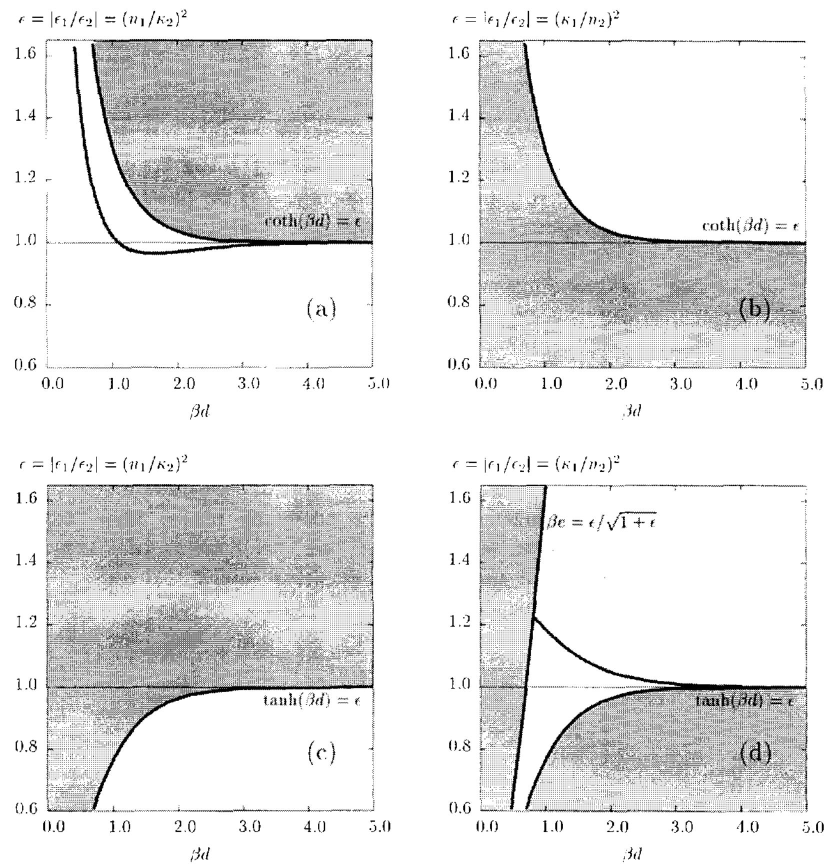

Fig. 2. Existence of F-TM modes and sign of the group velocity for: (a) the symmetrical F-TM mode of a metal-film waveguide (MFW); (b) the symmetrical F-TM mode of an optical strip-line (OSL); (c) the antisymmetrical F-TM mode of a MFW; (d) the antisymmetrical F-TM mode of an OSL. Dark regions correspond to forbidden modes, light grey regions are for positive group velocities and white regions are for negative velocities.

flow of the Poynting vector, along the direction of propagation z and per unit y width, inside the central layer, is given by

$$P_l = \int_0^d H_y E_x^* \, dx = \int_0^d H_y \left[\frac{i}{\omega \epsilon_2} \frac{\partial H_y}{\partial z} \right]^* dx, \qquad (7)$$

or using Eq. (1) or Eq. (3):

$$P_l = A^2 \frac{\beta d}{2\omega\epsilon_2} \left(\frac{\sinh(2\alpha_2 d)}{2\alpha_2 d} + 1 \right), \quad \text{(S) mode,}$$

$$P_l = A^2 \frac{\beta d}{2\omega\epsilon_2} \left(\frac{\sinh(2\alpha_2 d)}{2\alpha_2 d} - 1 \right), \quad \text{(AS) mode.} \qquad (8)$$

Likewise, the power flow of the Poynting vector inside the substrate is given by

$$P_s = \int_d^\infty H_y E_x^* \, dx = B^2 \frac{\beta d}{\omega\epsilon_1} \frac{\exp(-2\alpha_1 d)}{2\alpha_1 d}. \qquad (9)$$

As the dielectric constant of the metal is negative, the power inside the metal propagates along the z axis, but in the direction of negative z. Conversely, the dielectric constant of the dielectric medium is positive, and the power inside it propagates along the z axis, in the direction of positive z. The ratio $C_f = |P_l/P_s|$ of these power flows is

$$C_f = \left| \frac{\epsilon_1}{\epsilon_2} \right| \frac{\alpha_1 d}{\cosh^2(\alpha_2 d)} \left(\frac{\sinh(2\alpha_2 d)}{2\alpha_2 d} + 1 \right), \quad \text{(S) mode,}$$

$$C_f = \left| \frac{\epsilon_1}{\epsilon_2} \right| \frac{\alpha_1 d}{\sinh^2(\alpha_2 d)} \left(\frac{\sinh(2\alpha_2 d)}{2\alpha_2 d} - 1 \right), \quad \text{(AS) mode,} \qquad (10)$$

and measures the confinement of the power flow inside the central layer or inside the substrate. When C_f equals 1, both power flows have the same modulus but opposite signs, and the group and energy velocities equal 0. When C_f is larger than 1, the power flow is more confined in the central layer, and if this layer is a metal, the energy propagates along the z axis in the direction of negative z and the group velocity is negative. This happens for the symmetrical F-TM mode of a metal-film waveguide as shown on Fig. 2(a). When C_f is smaller than 1, the power flow is more confined in the substrate, and if the substrate is a metal, the group velocity is negative. This happens for the antisymmetrical F-TM mode of an optical strip-line waveguide as shown on Fig. 2(d).

In order to plot Figs. 2 to 6, we solved the implicit dispersion relations of Eq. (4), and then computed the phase and group velocities using Eq. (6). Note that Eq. (4) can have at most one solution in ω for a given β. In Fig. 2, the equations of the curves separating the regions of forbidden or existing modes were obtained analytically from Eq. (4), while the curves separating positive and negative group velocities were obtained numerically.

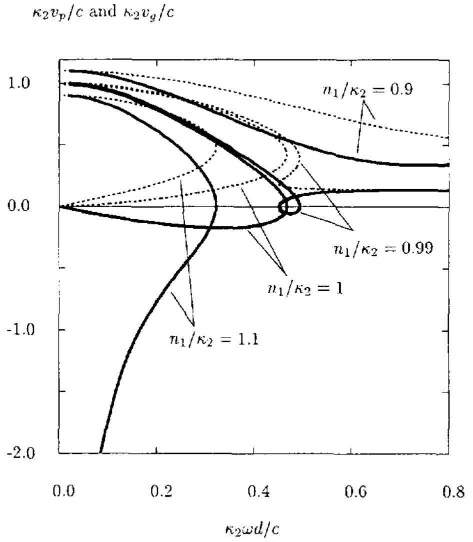

Fig. 3. Normalized phase velocity (dotted line) and group velocity (solid line) for the symmetrical F-TM mode of a metal-film waveguide.

Fig. 3 shows the phase velocity and the group velocity for the symmetrical mode of a metal-film waveguide ($\epsilon_2 < 0$) for several values of $n_1/\kappa_2 = \sqrt{|\epsilon_1/\epsilon_2|}$. It can be remarked that the group velocity becomes negative when

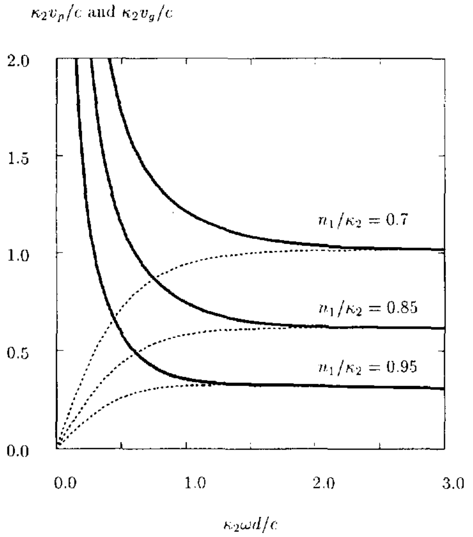

Fig. 4. Normalized phase velocity (dotted line) and group velocity (solid line) for the antisymmetrical F-TM mode of a metal-film waveguide.

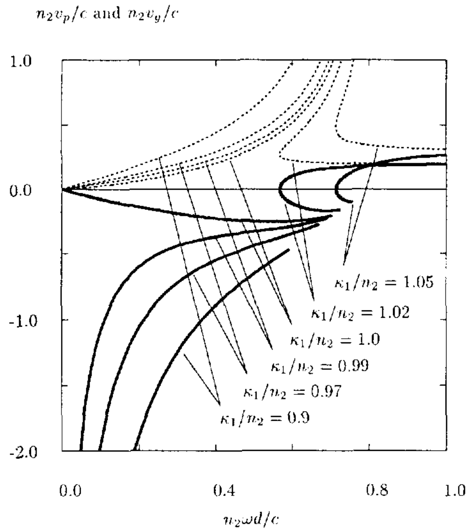

Fig. 5. Normalized phase velocity (dotted line) and group velocity (solid line) for the antisymmetrical F-TM mode of an optical strip-line waveguide.

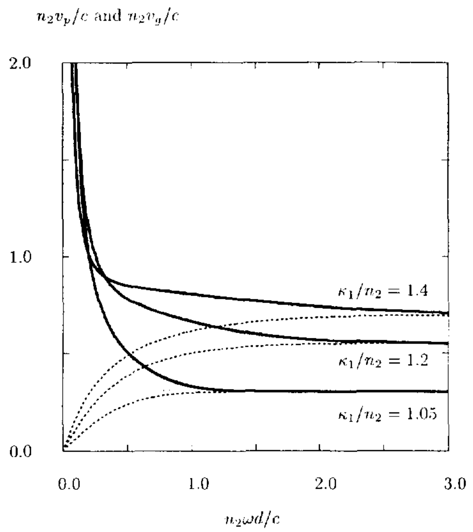

Fig. 6. Normalized phase velocity (dotted line) and group velocity (solid line) for the symmetrical F-TM mode of an optical strip-line waveguide.

n_1/κ_2 is greater than 0.97. Fig. 4 shows the same velocities for the antisymmetrical mode of a metal-film waveguide. The group velocity never becomes negative whatever the value of n_1/κ_2, as is also shown on Fig. 2(c). Fig. 5 shows the phase velocity and the group velocity for the antisymmetrical mode of an optical strip-line waveguide ($\epsilon_1 < 0$) for several values of $\kappa_1/n_2 = \sqrt{|\epsilon_1/\epsilon_2|}$. It can be remarked that the group velocity becomes negative when κ_1/n_2 is smaller than 1.22. Fig. 6 shows the same velocities for the symmetrical mode of an optical strip-line. The group velocity never becomes negative whatever the value of κ_1/n_2, as is also shown on Fig. 2(b).

Neglecting the ohmic losses, this phenomenon of negative group velocities should be observed for instance in waveguides made of lithium niobate $LiNbO_3$ ($n = 2.35$) or rutile TiO_2 ($n = 2.7$) associated with silver for wavelengths between 0.4 μm and 0.5 μm since $\kappa_{Ag} = 1.93$ at 0.4 μm and $\kappa_{Ag} = 2.87$ at 0.5 μm, or with gold for wavelengths between 0.5 μm and 0.6 μm since $\kappa_{Au} = 1.84$ at 0.5 μm and $\kappa_{Au} = 2.9$ at 0.6 μm.

Our prediction for negative group velocities is based purely on the application of Maxwell's equations. Further work is needed to fully understand the physical implications of negative group velocities, and thus this can not be treated in detail within this first article. The first question that arises is how can a laser beam be injected in such a negative group velocity waveguide? Classically, we need to phase-match the incident mode with the guided mode. Thus the direction of phase velocity of the guided frustrated TM mode will be the same as that of the exciting light beam. But as the energy is contrapropagating with the phase, light will exit the waveguide at the opposite side of what is usually expected (Fig. 7). A successful experiment of this type would be a neat demonstration of the existence of negative group velocities. Note that the experimental arrangement shown in Fig. 7 is rather more illustrative than realistic, since ohmic losses would probably be far to high in practice. A quite different approach to experimental confirmation is required indeed, and still needs to be found.

As a conclusion, we have shown that the symmetrical frustrated TM mode of a metal-film optical waveguide and the antisymmetrical frustrated TM mode of an optical

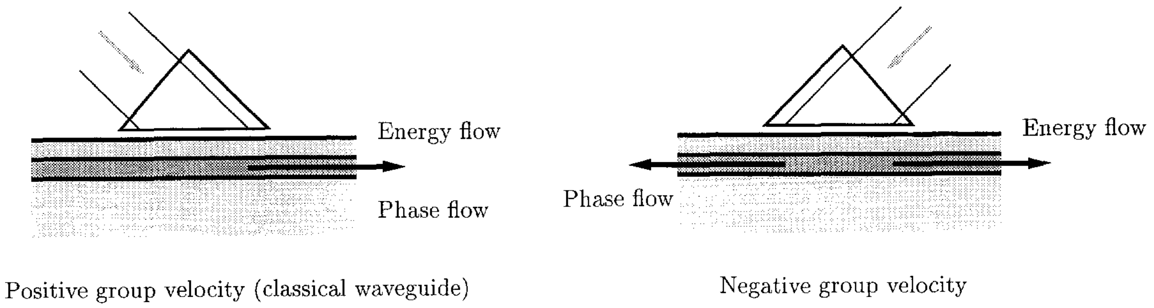

Fig. 7. Illustrative experimental arrangement using a prism coupler for the observation of negative group velocities.

strip-line waveguide have a negative group velocity depending on the ratio of the refractive index of the dielectric medium and the imaginary part of the refractive index of the metal. This phenomenon should be observed for instance in optical waveguides combining lithium niobate or rutile with gold or silver.

References

[1] P.K. Tien, Rev. Mod. Phys. 49 (1977) 361.
[2] P. Yeh, Optical waves in layered media (Wiley, New York, 1988).
[3] A. Yariv and P. Yeh, Optical waves in crystals (Wiley, New York, 1984).