# Nonlinear joint-transform correlation: an optimal solution for adaptive image discrimination and input noise robustness

**Philippe Refregier and Vincent Laude**

*Laboratoire Central de Recherches, Thomson-CSF, 91404 Orsay Cedex, France*

**Bahram Javidi**

*Department of Electrical Engineering, University of Connecticut, Storrs, Connecticut 06296-3157*

Received September 14, 1993

We develop a processor for pattern recognition that is optimum in terms of discrimination and is tolerant to variations of the object to be recognized. This optimum processor is found to be adaptive nonlinear joint-transform correlator.

Within the past decade different architectures for optical correlators have been demonstrated.[1-3] Although the spatial matched filter is optimal for noise robustness, its limitations, such as broad correlation peaks, sensitivity to distortion, and low discrimination capabilities,[4] are well known. As a result, different filters for optical correlation have been proposed.[4-9] On the other hand, nonlinear joint-transform correlators[3] (JTC's) have been shown to be discriminant with good correlation performance. For optical correlation, different criteria have been proposed to characterize the filter performances.[10] It has been shown[10] that some of the most interesting criteria are related to the noise robustness of the filter and the sharpness of the correlation function. Furthermore, the importance of finding trade-offs among different criteria is now well established.[8,10] It has been shown that this approach[8] leads to useful filters and figures of merit.

However, until now the discrimination capabilities of the filter have been optimized indirectly by minimization of either the sharpness of the correlation function[8] or the energy of the correlation function with false objects (that is, objects to be rejected) or background models to be discriminated against.[11] It is one of the main purposes of this Letter to determine an optimal method for discrimination capabilities of the processor that does not need *a priori* knowledge of the false objects or of the background. We show that this optimal processor has strong analogies with nonlinear JTC's.[3]

In the analysis, one-dimensional notations are used for simplicity with no loss of generality. Let $r(t)$ and $s(t)$, respectively; denote the reference and input images. The output of the optimum processor, $C(t) = \sum_{\xi=1}^{N} h(t+\xi)^* s(\xi)$, is the correlation between the input image and a filter function $h(t)$, where $*$ denotes complex conjugation and $N$ is the total number of pixels. In the derivation of the optimum processor, we will not restrict the filter function $h(t)$ to being linear.

When the input image is the same as the reference object, it is imposed that the filter produces the correlation peak $C_0 = C(0)$. This leads to the constraint

$$\sum_{t=1}^{N} h(t)^* r(t) = C_0 . \tag{1}$$

If the input image is a modified or a distorted version $r(t) + \delta r(t)$ of the reference image $r(t)$, one generally requires the variation in the correlation peak that is due to perturbation $\delta r(t)$ to be very small, that is, $\sum_t h(t)^*[r(t) + \delta r(t)] \simeq C_0$ or $\sum_t h(t)^* \delta r(t)] \simeq 0$. A classical method for achieving this requirement is to minimize the mean-squared value of the correlation with the perturbation $\delta r(t)$. In the Fourier domain, the correlation energy that is due to the perturbation is $\sum_k |\hat{h}(k)|^2 |\hat{\delta r}(k)|^2$, where $\hat{h}(k)$ and $\hat{\delta r}(k)$ denote the Fourier transform of $h(t)$ and $\delta r(t)$, respectively. With no *a priori* knowledge of perturbation $\delta r(t)$, let us consider that it is a zero-mean white random vector. The mean-squared value of $\sum_k |\hat{h}(k)|^2 |\hat{\delta r}(k)|^2$ is then

$$\sigma_m^2[h] = \sum_k |\hat{h}(k)|^2 \sigma_r^2 , \tag{2}$$

where $\sigma_r^2 = \langle |\delta r(t)|^2 \rangle$ and the angle brackets denote the mean value.

To optimize the discrimination capabilities of the processor, one may minimized the energy of the correlation function that is due to any input image $s(t)$. Because we consider Eq. (1) as a constraint, this is equivalent to minimizing:

$$E_s[h] = \sum_k |\hat{h}(k)|^2 |\hat{s}(k)|^2 , \tag{3}$$

where $\hat{s}(k)$ is the Fourier transform of $s(t)$.

The problem is now clearly defined: $\sigma_m^2[h]$ [Eq. (2)] and $E_s[h]$ [Eq. (3)] have to be minimized under the constraint of Eq. (1). The optimum processor corresponds to the optimal trade-off[8] between $\sigma_m^2[h]$ and $E_s[h]$. After appropriate substitution and taking into account that $C_0$ is arbitrary,[12] it can be shown that the optimum processor is

$$\hat{h}(k) = \frac{\hat{r}(k)}{\sigma^2 + |\hat{s}(k)|^2} , \tag{4}$$

0146-9592/94/060405-03$6.00/0 

where $\sigma^2$ is given positive constant. With Eq. (4), the Fourier transform of the correlation output is

$$\hat{C}(k) = \frac{\hat{r}(k)^*\hat{s}(k)}{\sigma^2 + |\hat{s}(k)|^2}. \qquad (5)$$

It is obvious from Eq. (5) that this optimal processor is a nonlinear filter, because it requires nonlinear transformation of the input image Fourier transform. The nonlinearity results from minimization of $E_s[h]$ in Eq. (3). This nonlinear processor is adaptive because the filter function is dependent on the input image energy spectrum.

The optimum nonlinear processor can be generalized when $\langle|\delta r(t)|^2\rangle$ has a power spectral density $\Delta(k)$. The nonlinear filter would then be $\hat{h}(k) = \hat{r}(k)/[\mu\Delta(k) + (1-\mu)|\hat{s}(k)|^2]$, where the parameter $\mu$ ($\mu \in [0; 1]$) is optimally balanced between noise robustness and discrimination capabilities.

It is interesting to note that if one replaces $\hat{s}(k)$ with $\hat{r}(k)$ in Eq. (4), the optimal trade-off filter for peak sharpness and noise robustness is obtained.[8] It can be shown that the optimal trade-off among input noise robustness $\sigma_m{}^2[h]$, discrimination $E_s[h]$, and correlation peak sharpness characterized by the correlation energy $\sum_k |\hat{h}(k)|^2|\hat{r}(k)|^2$ leads to the optimum processor: $\hat{h}(k) = \hat{r}(k)/[\sigma^2 + (1-\mu)|\hat{r}(k)|^2 + \mu|\hat{s}(k)|^2]$. For the particular value $\mu = 1/2$, the expression for the Fourier transform of the optimum nonlinear processor output is

$$\hat{C}(k) = \frac{\hat{r}(k)^*\hat{s}(k)}{2\sigma^2 + |\hat{r}(k)|^2 + |\hat{s}(k)|^2}. \qquad (6)$$

Equations (5) and (6) are similar to the Fourier transform of a nonlinear JTC output,[3] which can be implemented optically. Indeed, the nonlinear solution is similar to the first-order output of the nonlinear JTC. Both have the same Fourier phase, and the amplitude modulation requires nonlinear transformation of both Fourier magnitudes of the reference function and the input function. Similarities between the optimum solutions in Eqs. (5) and (6) and the nonlinear JTC are due to the optimization of the discrimination, which requires adaptive nonlinear transformations on the input image Fourier magnitude. Maximization of the peak sharpness requires nonlinear transformation on the reference image Fourier magnitude as in Eq. (6).

Let us now illustrate the performance of the optimum nonlinear processor with numerical simulations performed on images of 256 × 256 pixels with gray levels. The reference image $r(t)$ is a car shown in Fig. 1 in an array of 64 × 64 pixels. The input image is shown in Fig. 2 and contains the reference object placed both on the top left-hand side and in the center of the input. On the bottom right-hand side the reference object has been rotated by 7 deg. This composite image has been placed in presence of white noise with uniform probability density in [0; $a$] (the mean is thus $a/2$ and the variance is $a^2/12$). The noise is additive except within the object in the center, where it is spatially disjoint (or nonoverlapping).[13] As a result, the reference objects with overlapping noise are not clearly visible in Fig. 2 owing to the very low input signal-to-noise ratio (−7 dB).

Numerical experiments were performed with the nonlinear filter given by Eq. (5). Input and reference images were normalized such that their maximum values were equal to unity, and we chose $\sigma^2 = 1000$. The correlation function is shown in Fig. 3. The results are found to be not very sensitive to the particular value of $\sigma^2$ (acceptable correlation performance is obtained with $\sigma^2 = 1$). It can be seen from Fig. 3 that the optimal nonlinear filter in Eq. (4) has a robust behavior to reference object distortion and overlapping nonoverlapping input noise.

The same numerical experiment was performed with the following linear filters: optimal trade-off, phase-only, inverse, and matched. The best results

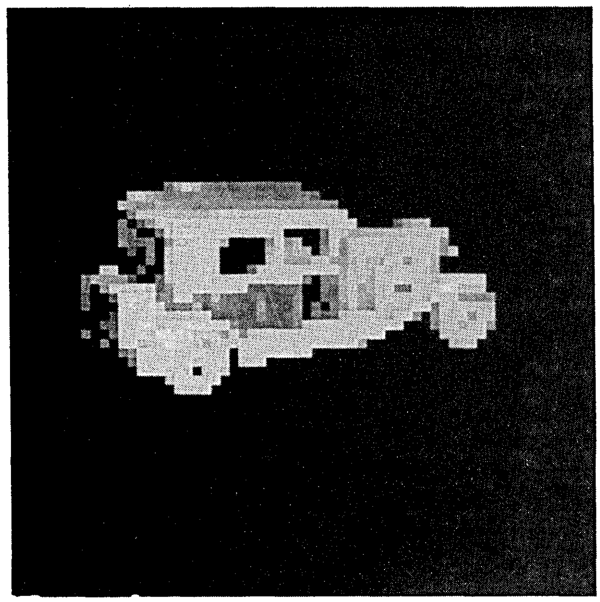

Fig. 1. Image of a car used as the reference object for numerical simulations.

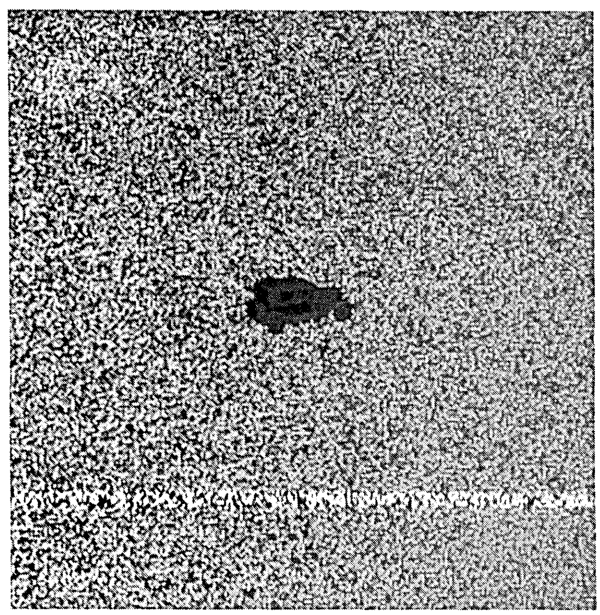

Fig. 2. Input image used for correlation tests.

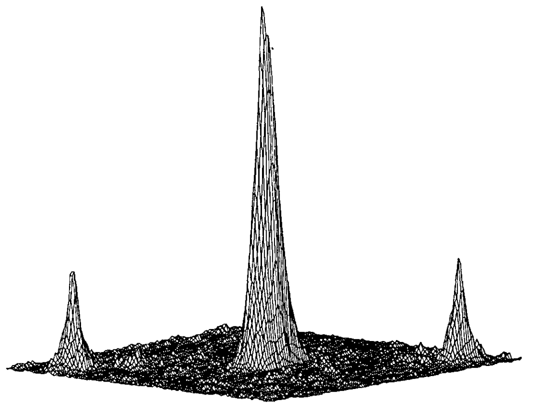

Fig. 3. Correlation function obtained with the proposed nonlinear filtering (nonlinear JTC) technique.

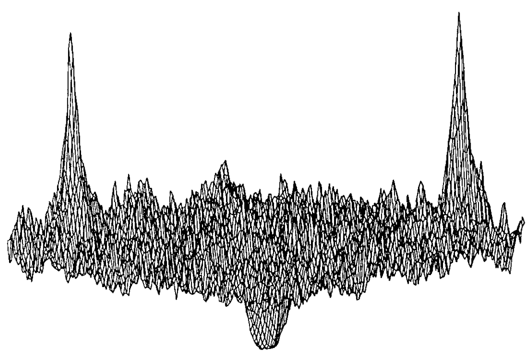

Fig. 4. Correlation function obtained with an optimal trade-off linear filter.

with linear filtering were obtained with an optimal trade-off filter, and its output correlation function is shown in Fig. 4. The optimal trade-off filter is obtained by replacing $s(t)$ in Eq. (4) with $r(t)$ and setting $\sigma^2 = 1$. The very low correlation peak value for the reference object in the center of the input image is due to the low mean value of the object in comparison with the mean value of the background noise, which is spatially disjoint with the object in the center.[13] The matched filter performance is very poor for the image used here.

The inverse filter and the phase-only filter produced only one detectable correlation peak corresponding to the reference object without rotation and with additive noise. Furthermore, as in Fig. 4, the correlation appears on a noisy background. These linear filters are too discriminant against the rotated reference object, and they are not discriminant enough against the input noise background. On the other hand, it can be seen from Fig. 3 that the nonlinear processor is very discriminant against noise background and tolerant to small rotations/distortions of the reference object.

In conclusion, we have designed a processor that is optimum in terms of discrimination and input noise robustness. This optimum processor is a nonlinear filter that can be implemented with a nonlinear JTC[3] and presents new theoretical insight into obtaining optimal nonlinear transformations. Computer simulations have illustrated the performance of the processor for noisy and distorted objects in the presence of both overlapping and nonoverlapping input noise. Further studies are necessary to characterize the performance of the proposed nonlinear processor.

The authors are grateful to J.-P. Huignard for his support in this research.

## References


1. A. Vander Lugt, IEEE Trans. Inform. Theory **IT-10,** 139 (1964).
2. C. S. Weaver and J. W. Goodman, Appl. Opt. **5,** 1248 (1966).
3. B. Javidi, Appl. Opt. **28,** 2358 (1989).
4. D. Casasent and D. Pasaltis, Appl. Opt. **15,** 1795 (1976).
5. H. J. Caulfield and W. T. Maloney, Appl. Opt. **8,** 2354 (1969).
6. J. L. Horner, Appl. Opt. **21,** 4511 (1982).
7. Y. N. Hsu and H. H. Arsenault, Appl. Opt. **21,** 4016 (1982).
8. Ph. Réfrégier, Opt. Lett. **15,** 854 (1990).
9. Ph. Réfrégier, Opt. Lett. **16,** 829 (1991).
10. B. V. K. Vijaya Kumar and L. Hassebrook, Appl. Opt. **29,** 2997 (1990).
11. L. P. Yaroslavsky, Appl. Opt. **31,** 1677 (1992).
12. To find the optimal trade-off between the two functions $\sigma_m{}^2[h]$ [Eq. (2)] and $E_s[h]$ [Eq. (3)] under the constraint of Eq. (1), one needs to minimize[8]: $\Psi[h] = \sigma_m{}^2[h] + \mu E_s[h] - \lambda \sum_{t=1}^{N} h(t)^* r(t)$. Setting $[\partial/\partial \hat{h}(k)]\Psi[h] = 0$ leads to $\hat{h}(k) = \lambda \hat{r}(k)/[\sigma_r{}^2 + \mu|\hat{s}(k)|^2]$, which can be written as $(\lambda/\mu)\hat{r}(k)/[\sigma^2 + |\hat{s}(k)|^2]$ with $\sigma^2 = \sigma_r{}^2/\mu$. Since $C_0$ is an arbitrary constraint, it can be chosen such that $\mu = \lambda$. One then obtains the filter of Eq. (4). It is necessary to emphasize that $\sigma^2$ is a parameter chosen for optimal balance between noise robustness and discrimination.
13. B. Javidi and J. Wang, Appl. Opt. **31,** 6826 (1992).