Spatial fluctuations of an optical field modulated with spatial light modulators and noisy input signals

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Received July 13, 1994; revised manuscript received November 14, 1994; accepted December 21, 1994

Electrically addressed spatial light modulators (EASLM's) are currently used in many coherent optical signal processing applications. Within the EASLM the input signal is generally transformed in a nonlinear fashion. The noise on the input signal that is sent to the EASLM is then also transformed nonlinearly. In particular, even with additive and homogeneous input noise, the color of the noise as well as the signal-to-noise ratio of the input signal can be significantly modified. We propose a new general formalism for the determination of the spatial covariance of the optical field that is due to nonlinear transformation of the input noise. Applications of this new formalism are illustrated for Gaussian correlated noise.

1. INTRODUCTION

Coherent optical signal processing for image processing, pattern recognition, correlation, and neural networks has been a subject of investigations for more than 20 years.[1–4] Constraints of optical implementation have important consequences on the capacities of the performed operation. It is clear that, in general, a classical spatial light modulator (SLM) will code the input information (the input signal) on a coherent beam as a complex function. For example, this is particularly the case with twisted nematic SLM's, for which each pixel acts on the incident light through the birefringence effect. Thus the information is not simply a real value for each pixel, as generally considered in numerical simulations, but a complex function (i.e., the phase and the modulus of the optical wave are dependently variable). From a pure signal point of view, this means that the performed operation is not the expected classical linear operation but a modified version. An analogous situation occurs for wave-front compensation with SLM's such as deformable mirror SLM's.

Let x denote the spatial coordinate of a pixel in the input signal (in general, an input image). Without loss of generality and in order to simplify the notations, we will consider one-dimensional notation in what follows. Let $f(x)$ denote the input signal (or image) that corresponds to the voltage sent to the electrically addressed spatial light modulator (EASLM), and let $\Psi(x)$ be the optical field after passage through the EASLM. Figure 1 sketches the general problem that we consider here, which corresponds to optical modulation with a SLM.

It is a classical and an important task in signal processing to characterize the statistical properties of the noise in order to be able to predict or quantify precisely the deviations from the ideal mathematical linear operation. Indeed, the characterization of the influence of the input modulation on the optical processing is important, since it can condition practical realizations.

The main goal of this paper is thus to characterize the random optical field when the input signal sent to the EASLM is corrupted by additive noise. More precisely, the first moment (mean value) and the second moment (correlation function) of the optical field will be determined.

This situation is illustrated in Fig. 2, which sketches the modification of the probability density of an input noise for a modulation law of the form $M(z) = z \exp(iKz)$. It will be seen below that such a modulation corresponds to realistic modulators.

In Fig. 3, the modulation of the spatial correlation of the fluctuation of the optical field is illustrated when a constant signal with noise is sent to a spatial modulator with a modulation law of the form $M(z) = z \exp(iKz)$.

Few studies[5–8] have been made about the influence of input image coding in optical correlation operations. In a previous paper a first analysis of the influence of input noise was performed in this context for binary images.[9] This research has been extended in Ref. 10 to gray-level images. It was shown that the input modulation can have strong effects on the noise robustness of the optical correlation. However, in Refs. 9 and 10, only the case of white Gaussian additive input noise was considered with three typical modulations.

In contrast with the approach of Ref. 10, we will here insist on the theoretical analysis and propose a new general formalism. We consider general additive input noise (i.e., not necessarily white and Gaussian), but special attention will be given to homogeneous input noise (homogeneity for a random function means that it is stationary in space; see, e.g., Ref. 11). Furthermore, modulation schemes more general than that in Ref. 10 can be described with our new formalism. Our goal is clearly to describe quantitatively, with a theoretical analysis, the profound modifications of the noise statistics (nonlinearity, nonhomogeneity, and modification of the color of the noise) that are due to phase modulation.

2. MODELING OF ELECTRICALLY ADDRESSED SPATIAL LIGHT MODULATOR TRANSMISSION

In general, in numerical studies of optical correlation a pure amplitude modulation is assumed, which is defined by $\Psi_A(x) = f(x)$. However, this modulation is, in

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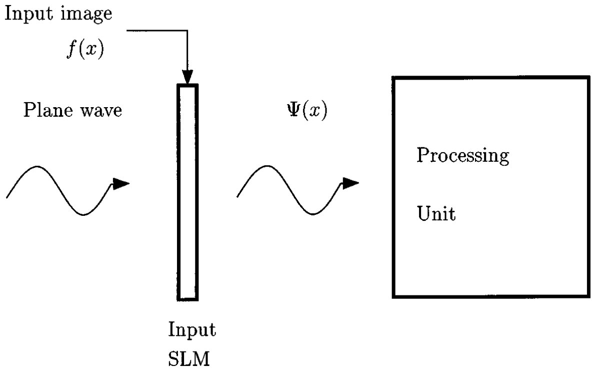

Fig. 1. Coding scheme of an input image with an EASLM. $f(x)$ denotes the input signal sent to the EASLM, and $\Psi(x)$ is the optical field after passage through the EASLM.

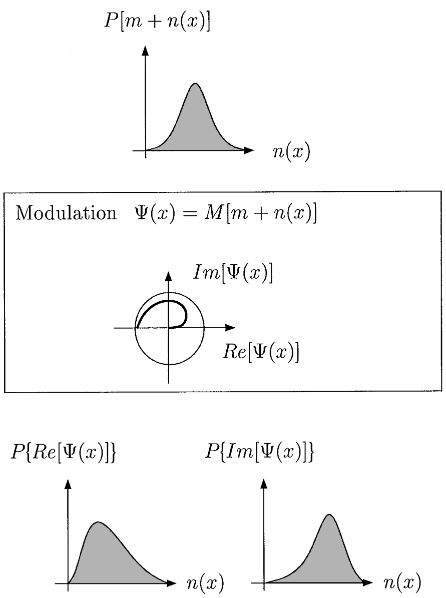

Fig. 2. Modification of the probability density of an input noise $n(x)$ when the modulation law is of the form $M(z) = z\ \exp(iKz)$. m is the signal without noise and Re( )[Im( )] indicates the real part (imaginary part) of a complex number.

practice, very difficult to achieve optically. On the other hand, pure phase modulation can be simply obtained with, for example, either a perfect deformable mirror SLM or a birefringent nematic crystal SLM. In that case the modulation is $\Psi_P(x) = \exp[iKf(x)]$, where K is the maximum phase shift. In recent papers[12,13] we showed that, for obtaining contrast maximum with a twisted nematic EASLM, the phase dependency can be approximated by a linear function of the imposed amplitude modulation. This result clearly demonstrated that a good approximation of the modulation performed by this EASLM can be modeled as the product of a perfect amplitude modulation and a linear phase shift: $\Psi_S[f(x)] = f(x)\exp[iKf(x)]$, where K is again the maximum phase shift.

Here we will consider a more general situation. Let the nonlinear relation between the input signal $f(x)$ sent to the EASLM and the optical field $\Psi(x)$ after passage through the EASLM be denoted by

$$\Psi(x) = M[f(x)]. \tag{1}$$

A generalization of the modulation schemes mentioned above can be described by the polynomial model

$$M(z) = \sum_{n=0}^{p} a_n z^n \exp(iKz). \tag{2}$$

Appendix A below discusses the particularities of this model.

It can be remarked that only amplitude modulation is linear. Indeed, $f(x) \rightarrow 2f(x)$ does not imply that $\Psi(x) \rightarrow 2\Psi(x)$ with general coupled amplitude/phase modulation.

In order to obtain tractable expressions, we will write the input modulation of Eq. (2) as a linear differential operator $\mathcal{A}$ applied to $\exp(iKz)$. More precisely, introducing

$$\mathcal{A} = \sum_{n=0}^{p} \frac{a_n}{i^n} \frac{\partial^n}{\partial K^n}, \tag{3}$$

with $i^2 = -1$, we can write $M(z)$ in Eq. (2) as

$$M(z) = \mathcal{A}\ \exp(iKz). \tag{4}$$

$\mathcal{A}$ will thus be called the generating differential operator of the modulation or, more briefly, the generating operator.

For example, the generating operator of pure phase modulation is clearly the identity. For coupled amplitude modulation as discussed above {i.e., $\Psi_S(x)[f(x)] = f(x)\exp[iKf(x)]$} the generating operator is $\mathcal{A} = (1/i)(\partial/\partial K)$. For pure amplitude modulation $[\Psi_A(x) = f(x)]$ the generating operator is $\mathcal{A} = (1/i)(\partial/\partial K)$ evaluated at $K = 0$.

3. RANDOM OPTICAL FIELD CHARACTERIZATION

A. Mean and Covariance of the Optical Field

A general task of signal processing is to process noisy input signals. This is clearly the case, for instance, for

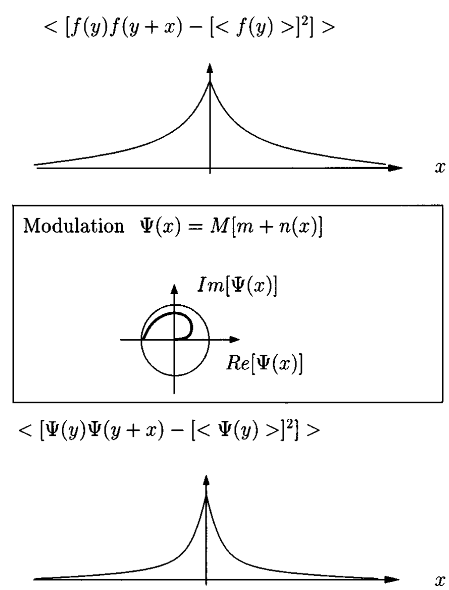

Fig. 3. Modification of the correlation function of the noise that is due to a modulation of the form $M(z) = z\ \exp(iKz)$. $f(x)$ is the input image, $\Psi(x)$ is the output optical field, and $\langle\rangle$ is the ensemble average operator. This example corresponds to a decorrelation.

optical correlation or for neural networks. Other sources of input noise can be the presence of noise on the driving signal or on the EASLM itself (for example, flicker noise in liquid-crystal SLM's). Another well-known situation is obtained for wave-front compensation, since the driving input signal is estimated from experimental measures. It is then known with an estimation error that may be modeled as additive input noise.

The goal of this section is to derive a model in order to describe the effect on the optical field $\Psi(x)$ of an input additive noise on $f(x)$. More precisely, the mean and the covariance function of $\Psi(x)$ will be determined as a function of the input noise characteristics.

If the input signal $f(x)$ is corrupted by an additive input noise [i.e., $f(x) \rightarrow f(x) + n(x)$], the optical field $\Psi(x)$ is modified in a nonlinear fashion:

$$\Psi(x) = M[f(x) + n(x)]. \tag{5}$$

In order to determine the mean and the covariance of the random field $\Psi(x)$, one needs to know the probability density $P[n(x)]$ as well as the joint probability density $P[n(x), n(y)]$ of the noise.

Let $\langle\,\rangle$ denote the ensemble average. Thus the mean value of the optical field after the EASLM is

$$\begin{aligned}\langle\Psi(x)\rangle &= \langle M[f(x) + n(x)]\rangle \\ &= \int M[f(x) + n(x)]P[n(x)]\mathrm{d}n(x).\end{aligned} \tag{6}$$

The covariance function of $\Psi(x)$ is defined by

$$C_{\Psi,\Psi}(x, y) = \langle[\delta\Psi(y)]^{*}\delta\Psi(x)\rangle, \tag{7}$$

where z^{*} means the complex-conjugate value of z and

$$\delta\Psi(x) = \Psi(x) - \langle\Psi(x)\rangle. \tag{8}$$

Using the generating operator of Eqs. (3) and (4), one can easily determine $\langle\Psi(x)\rangle$ and $C_{\Psi,\Psi}(x, y)$. For this purpose we introduce the notation

$$\mathcal{A}_1 = \sum_{n=0}^{p} \frac{a_n}{i^n} \frac{\partial^n}{\partial {K_1}^n}, \tag{9}$$

and we summarize our results with the following property.

Property 1. The mean and the covariance function of $\Psi(x)$ are completely determined with the first and second characteristic functions of the noise. The mean is

$$\langle\Psi(x)\rangle = \mathcal{A}\{\exp[iKf(x)]G_x(K)\}, \tag{10}$$

where $G_x(K)$ is the first characteristic function of $P[n(x)]$:

$$G_x(K) = \langle\exp[iKn(x)]\rangle. \tag{11}$$

The covariance function is

$$\begin{aligned}C_{\Psi,\Psi}(x, y) = \mathcal{A}_1{\mathcal{A}_2}^{*}\{&\exp[iK_1f(x) - iK_2f(y)] \\ &\times \Delta G_{x,y}(K_1, K_2)\}|_{K_1=K_2=K},\end{aligned} \tag{12}$$

where the notation $|_{K_1=K_2=K}$ means that the final expression must be evaluated at $K_1 = K_2 = K$, where

$$\Delta G_{x,y}(K_1, K_2) = G_{x,y}(K_1, K_2) - G_x(K_1)G_y(K_2)^{*} \tag{13}$$

and $G_{x,y}(K_1, K_2)$ is the second characteristic function of $P[n(x), n(y)]$:

$$G_{x,y}(K_1, K_2) = \langle\exp[iK_1n(x) - iK_2n(y)]\rangle. \tag{14}$$

The first and second characteristic function can also be written as

$$G_x(K) = \int \exp[iKn(x)]P[n(x)]\mathrm{d}n(x), \tag{15}$$

$$\begin{aligned}G_{x,y}(K_1, K_2) = \iint &\exp[iK_1n(x) - iK_2n(y)] \\ &\times P[n(x), n(y)]\mathrm{d}n(x)\mathrm{d}n(y).\end{aligned} \tag{16}$$

This property can be shown as follows:

$$\langle\Psi(x)\rangle = \langle\mathcal{A}(\exp\{iK[f(x) + n(x)]\})\rangle. \tag{17}$$

Only $\exp[iKn(x)]$ is dependent on the noise, and since $\mathcal{A}$ is linear, we have

$$\langle\Psi(x)\rangle = \mathcal{A}\{\exp[iKf(x)]\langle\exp[iKn(x)]\rangle\}. \tag{18}$$

Furthermore,

$$\begin{aligned}[\Psi(y)]^{*}\Psi(x) = \mathcal{A}_1&\{\exp[iK_1f(x) + iK_1n(x)]\} \\ &\times \mathcal{A}_2^{*}\{\exp[iK_2f(y) + iK_2n(y)]\}^{*}.\end{aligned} \tag{19}$$

Here again, only $\exp[iK_1n(x) - iK_2n(y)]$ is dependent on the noise, and since $\mathcal{A}_1$ and $\mathcal{A}_2$ are linear, it follows that

$$\begin{aligned}\langle[\Psi(y)]^{*}\Psi(x)\rangle = \mathcal{A}_1\mathcal{A}_2^{*}\{&\exp[iK_1f(x) - iK_2f(y)] \\ &\times \langle\exp[iK_1n(x) - iK_2n(y)]\rangle\},\end{aligned} \tag{20}$$

and property 1 is obtained with Eqs. (7) and (8). ■

This property shows that the modeling of the modulation with the generating operator of Eq. (3) leads to very simple expressions for the moments of $\Psi(x)$ [Eqs. (10) and (12)]. Indeed, we have shown that the first two moments of $\Psi(x)$ can be deduced from the characteristic functions of the noise of first and second orders.

B. Case of Homogeneous Input Noise

Let us assume that the input noise is strictly homogeneous. In other words,

$$P[n(x),\, n(y)] = P[n(x-y),\, n(0)]\,. \tag{21}$$

For the first and second moments we also have

$$\langle n(x)\rangle = m_n\,, \tag{22}$$

where m_n is independent of x and

$$\langle n(x)n(x+y)\rangle - {m_n}^2 = \Gamma_y\,, \tag{23}$$

m_n is the mean value of the input noise, and Γ_y is the covariance function of the input noise. The first and second characteristic functions satisfy

$$G_x(K) = G(K)\,, \tag{24}$$

$$G_{x,y}(K_1,\, K_2) = G_{x-y}(K_1,\, K_2)\,. \tag{25}$$

In other words, $G_x(K)$ is independent of x and $G_{x,y}$ is dependent on only $x - y$.

Thus, using property 1, we can see that the mean value $\langle\Psi(x)\rangle$ is dependent on x, and this result is analogous to amplitude modulation, since $\langle f(x) + n(x)\rangle$ is also dependent on x. Then, in order to avoid characterizing a random field as nonhomogeneous as a result of this trivial problem, we consider the random field $\delta\Psi(x)$. However, even in that case, $C_{\Psi,\Psi}(x,\, y)$ is not a function of only $x - y$, which means that the random field $\delta\Psi(x)$ is not homogeneous, although it is generated by a homogeneous input noise. Indeed,

$$\begin{aligned} C_{\Psi,\Psi}(x,\, y) = {}& \mathcal{A}_1\mathcal{A}_2^*\{\exp[iK_1 f(x) - iK_2 f(y)] \\ & \times G_{x-y}(K_1,\, K_2) - G(K_1)G(K_2)^*\}|_{K_1=K_2=K}\,. \end{aligned} \tag{26}$$

The term $\exp[iK_1 f(x) - iK_2 f(y)]$ in this expression shows that, in general, $C_{\Psi,\Psi}(x,\, y)$ cannot be written as $C_{\Psi,\Psi}(x - y)$. This nonconservation of the homogeneity comes from the nonlinear coupling between $f(x)$ and $n(x)$, which results from the modulation $M[f(x)]$.

However, spatially independent noises defined by the property

$$P[n(x),\, n(y)] = P[n(x)]P[n(y)] \tag{27}$$

lead to the following property.

Property 2. If the input noise $n(x)$ is homogeneous and spatially independent, then the output field $\delta\Psi(x)$ is white and its variance is given by

$$\begin{aligned} C_{\Psi,\Psi}(x,\, x) = {}& \mathcal{A}_1\mathcal{A}_2^*\{\exp[i(K_1 - K_2)f(x)] \\ & \times G(K_1 - K_2) - G(K_1)G(K_2)^*\}|_{K_1=K_2=K}\,. \end{aligned} \tag{28}$$

This relation can be proved, noting that, if $x \neq y$, then

$$\begin{aligned} G_{x,y}(K_1,\, K_2) = \iint & \exp[iK_1 n(x) - iK_2 n(y)] \\ & \times P[n(x)]P[n(y)]\mathrm{d}n(x)\mathrm{d}n(y) \end{aligned} \tag{29}$$

and thus

$$G_{x,y}(K_1,\, K_2) = G(K_1)G(K_2)^*\,. \tag{30}$$

Hence, with Eq. (12), one can see that $C_{\Psi,\Psi}(x,\, y) = 0$. On the other hand, if $x = y$, then

$$P[n_1(x),\, n_2(x)] = P[n_1(x)]\delta[n_1(x) - n_2(x)]\,. \tag{31}$$

We can then deduce that

$$G_{x,x}(K_1,\, K_2) = G(K_1 - K_2)\,, \tag{32}$$

where $G(K_1 - K_2)$ is now the first-order characteristic function; this last result proves the property. ■

4. EXAMPLES OF APPLICATIONS: GAUSSIAN NOISE

A. Introduction

We now propose to illustrate the application of the above method to the determination of the mean and the covariance function of $\Psi(x)$ for a colored Gaussian input noise and for linearly coupled amplitude phase modulation and pure phase modulation. These cases generalize the results given in Ref. 10, in which only the case of homogeneous white Gaussian input noise was analyzed.

Let N denote the number of pixels of the EASLM and of the input signal. A Gaussian noise can be characterized by the probability density

$$\begin{aligned} & P[n(0),\, n(1),\, \ldots,\, n(N-1)] \\ & \quad = A_0 \exp\left[-\frac{1}{2}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} n(x){\Gamma_{x,y}}^{-1}n(y)\right], \end{aligned} \tag{33}$$

where A_0 is a normalization constant.

The characteristic function of first order is equal to

$$G_x(K) = \exp(-{}^1\!/\!_2\Gamma_{x,x}K^2)\,. \tag{34}$$

It can also be shown with Gaussian integrals that the characteristic function of second order is

$$G_{x,y}(K_1,\, K_2) = \exp(-{}^1\!/\!_2\Gamma_{x,x}{K_1}^2 - {}^1\!/\!_2\Gamma_{y,y}{K_2}^2 + \Gamma_{x,y}K_1K_2)\,. \tag{35}$$

B. Linearly Coupled Amplitude Phase Modulation

1. *Correlation of the Optical Field Fluctuations*

Let us consider the example of a linearly coupled amplitude phase modulation $\Psi_S[f(x)] = f(x)\exp[iKf(x)]$; the generating operator is thus $\mathcal{A} = (1/i)(\partial/\partial K)$.

The first moment of $\Psi(x)$ is then

$$\langle\Psi(x)\rangle = -i\,\frac{\partial}{\partial K}\,\exp[iKf(x) - {}^1\!/\!_2\Gamma_{x,x}K^2]\,. \tag{36}$$

Thus

$$\langle\Psi(x)\rangle = [f(x) + i\Gamma_{x,x}K]\exp[iKf(x) - {}^1\!/\!_2\Gamma_{x,x}K^2]\,. \tag{37}$$

It can be shown (see Appendix B below) that the second moment $C_{\Psi,\Psi}(x,\, y)$ is

$$C_{\Psi,\Psi}(x, y) = [E(x, y) - F(x, y)]L(x, y), \quad (38)$$

with

$$E(x, y) = \{[if(x) - \Gamma_{x,x}K + \Gamma_{x,y}K][-if(y) - \Gamma_{y,y}K + \Gamma_{y,x}K] + \Gamma_{y,x}\}\exp(\Gamma_{x,y}K^2),$$
$$F(x, y) = [f(x) + i\Gamma_{x,x}K][f(y) - i\Gamma_{y,y}K^2],$$
$$L(x, y) = \exp\{iK[f(x) - f(y)] - {}^1\!/\!_2K^2(\Gamma_{x,x} + \Gamma_{y,y})\}. \quad (39)$$

2. *Modification of the Color of the Noise*

Let us consider the simple case of a constant signal, $f(x) = m$, with a homogeneous input noise. In that case the covariance of the spatial fluctuations of the optical field is homogeneous and can be written as $C_{\Psi,\Psi}(x, y) = C_{\Psi,\Psi}(x - y)$. We then have

$$E(x, y) = [m^2 + K^2(\Gamma_{x-y} - \Gamma_0)^2 + \Gamma_{x-y}]\exp(\Gamma_{x-y}K^2),$$
$$F(x, y) = m^2 + {\Gamma_0}^2K^2,$$
$$L(x, y) = \exp(-\Gamma_0 K^2). \quad (40)$$

In what follows, x will be used instead of $x - y$ for reasons of simplicity. In order to study the modification of the color of the noise, we introduce the factor ρ defined below, which permits one to compare the correlation of the input signal with that of the optical field. The correlation coefficient of the noise on the input signal is $\gamma(x) = \Gamma_x/\Gamma_0$. The correlation coefficient of the spatial fluctuations of the optical field is $\beta(x) = C_{\Psi,\Psi}(x)/C_{\Psi,\Psi}(0)$. The coefficient ρ is then defined by $\rho = \beta(x)/\gamma(x)$, or, in other words,

$$\rho = \frac{\Gamma_0}{\Gamma_x}\frac{C_{\Psi,\Psi}(x)}{C_{\Psi,\Psi}(0)}. \quad (41)$$

In order to simplify the analysis, one can introduce the reduced variables

$$a = K^2\Gamma_0, \qquad \mu = m^2/\Gamma_0; \quad (42)$$

μ is thus the input signal ratio. From Eq. (41) it is not difficult to deduce the mathematical expression for ρ:

$$\rho = \frac{1}{\gamma}\frac{[a(\gamma - 1)^2 + \mu + \gamma]\exp(a\gamma) - (\mu + a)}{(1 + \mu)\exp(a) - (\mu + a)}. \quad (43)$$

The definition of γ implies that $-1 \leq \gamma \leq 1$. So, in order to study the modification of the noise color, from Eq. (43) we see that it is sufficient to study ρ as a function of γ. For a given value of γ, if ρ is equal to 1, the coding does not modify the color. If $|\rho| \leq 1$, the spatial fluctuations are decorrelated; on the other hand, if $\rho \geq 1$, their correlation is increased. With $K = 0$ we find the straightforward result of pure amplitude modulation.

In Fig. 4, ρ is plotted as a function of γ for different values of a with $\mu = 0$ (i.e., $m = 0$). We can see that, except for $a = 0$ (i.e., $K = 0$), $|\rho|$ is always smaller than 1. This means that the optical coding decorrelates the input noise. In Fig. 5 the same curves are plotted but for $\mu = 1$. We obtain globally the same conclusion. One can thus conjecture that a linear coupled phase modulation will necessarily decorrelate the input noise in comparison with a pure amplitude coding. Furthermore, the decorrelation is higher not only if K increases but also if the input noise increases. However, this decorrelation decreases if μ increases.

3. *Modification of the Signal-to-Noise Ratio*

Let us now analyze the transformation of the power of the noise or, more precisely, the output signal-to-noise ratio as a function of the input signal-to-noise ratio. We consider now $f(x) \neq 0$, and we study $C_{\Psi,\Psi}(x, x)$. One has

$$E(x, x) = [f^2(x) + \Gamma_0]\exp(\Gamma_0 K^2),$$

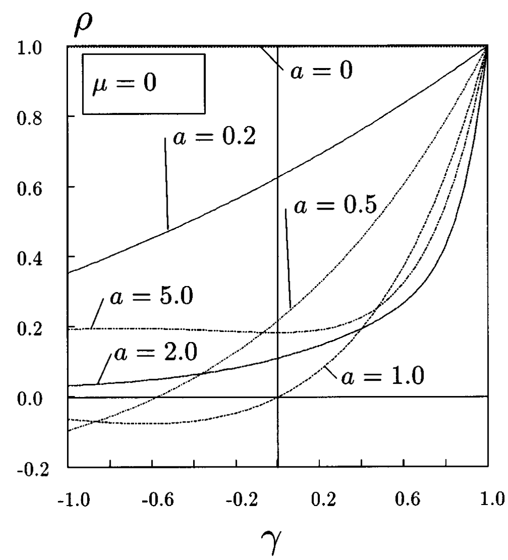

Fig. 4. ρ (which is the ratio of the correlation coefficient of the spatial fluctuations of the optical field versus the ratio of the correlation coefficient of the input noise) is plotted as a function of γ (the correlation coefficient of the input noise) for different values of a with $\mu = 0$ (i.e., $m = 0$).

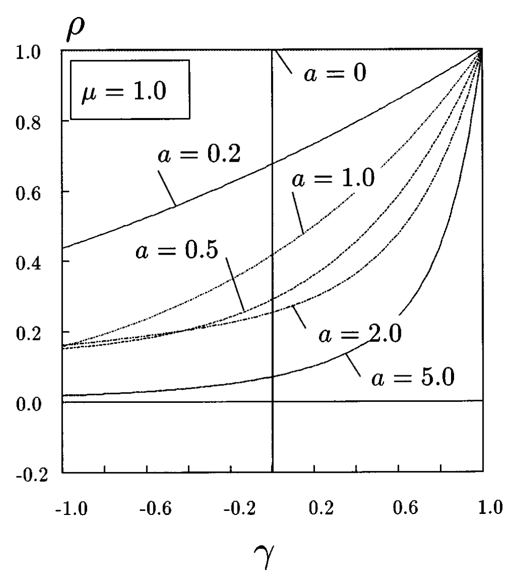

Fig. 5. ρ (which is the ratio of the correlation coefficient of the spatial fluctuations of the optical field versus the ratio of the correlation coefficient of the input noise) is plotted as a function of γ (the correlation coefficient of the input noise) for different values of a with $\mu = 1$.

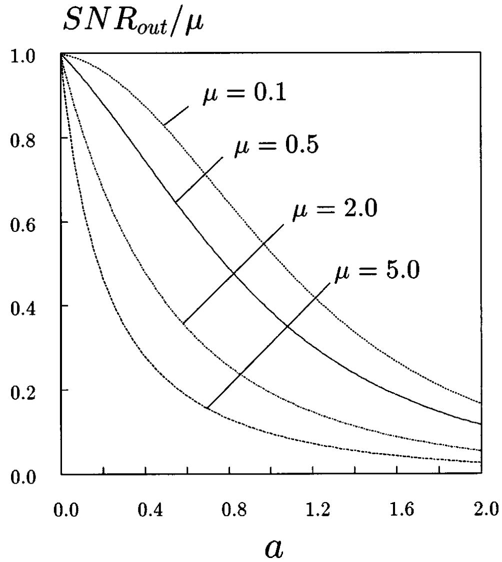

Fig. 6. Ratio of output signal-to-noise ratio to input signal-to-noise ratio as a function of a for different values of μ.

$$F(x, x) = [f^2(x) + {\Gamma_0}^2 K^2],$$
$$L(x, x) = \exp(-\Gamma_0 K^2), \tag{44}$$

and thus

$$C_{\Psi,\Psi}(x, x) = [f^2(x) + \Gamma_0] - [f^2(x) + {\Gamma_0}^2 K^2]\exp(-\Gamma_0 K^2). \tag{45}$$

Furthermore,

$$|\langle\Psi(x)\rangle|^2 = [f^2(x) + {\Gamma_0}^2 K^2]\exp(-\Gamma_0 K^2). \tag{46}$$

In order to obtain a meaningful signal-to-noise ratio, one must define a useful signal. For that purpose let us consider the modulus squared of the mean value of $\Psi(x)$ when there is no signal [i.e., when $f(x) = 0$]. We write this value as $|\langle\Psi_0(x)\rangle|^2$. We thus define the useful signal as

$$\Delta|\langle\Psi(x)\rangle|^2 = |\langle\Psi(x)\rangle|^2 - |\langle\Psi_0(x)\rangle|^2, \tag{47}$$

and the output signal-to-noise ratio is thus

$$\mathrm{SNR}_{\mathrm{out}} = \frac{\Delta|\langle\Psi(x)\rangle|^2}{C_{\Psi,\Psi}(x, x)}. \tag{48}$$

With the reduced variables one obtains

$$\mathrm{SNR}_{\mathrm{out}} = \frac{\mu}{(1 + \mu)\exp(a) - (\mu + a)}, \tag{49}$$

with $\mu = f^2(x)/\Gamma_0$ (i.e., the input signal-to-noise ratio). If $K = 0$, one obtains the input signal-to-noise ratio.

In Fig. 6, $\mathrm{SNR}_{\mathrm{out}}/\mu$ is plotted as a function of a for different values of μ. One can see that an increase of a (i.e., in K) decreases the signal-to-noise ratio. Furthermore, $\mathrm{SNR}_{\mathrm{out}}/\mu$ decreases as μ increases, which means that the deterioration of the signal-to-noise ratio is worse when the input SNR increases. It is very easy to analyze this result, since

$$\frac{\mathrm{SNR}_{\mathrm{out}}}{\mu} = \frac{1}{\exp(a) - a + \mu[\exp(a) - 1]}, \tag{50}$$

which is a decreasing function of μ and a decreasing function of a. At the limit of small input SNR (i.e., small μ) one obtains

$$\frac{\mathrm{SNR}_{\mathrm{out}}}{\mu} \simeq \frac{1}{\exp(a) - a}. \tag{51}$$

This is a positive result, since, at low μ, which corresponds to the most critical practical situations, the decrease of the SNR is linear with a factor that can be determined very easily.

C. Pure Phase Modulation

Let us now analyze the case of pure phase modulation for homogeneous Gaussian input noise with the same assumptions as those in Subsection 4.B.2 above. One has

$$G_x(K) = \exp(-{}^1\!/\!{}_2\Gamma_0 K^2), \tag{52}$$

$$\Delta G_{x,y}(K, K) = \exp(-\Gamma_0 K^2 + \Gamma_{x-y} K^2) - \exp(-\Gamma_0 K^2). \tag{53}$$

With the reduced variables defined above, one obtains

$$C_{\Psi,\Psi}(x) = \exp(-a)\{\exp[a\gamma(x)] - 1\}, \tag{54}$$

and thus

$$\rho = \frac{1}{\gamma(x)}\,\frac{\exp[a\gamma(x)] - 1}{\exp(a) - 1}. \tag{55}$$

It is easy to check that $\rho = 1$ if $x = 0$ [since, in that case, $\gamma(x) = 1$]. In order to study ρ as a function of γ, we analyze its series expansion:

$$\rho = \frac{a}{\exp(a) - 1}\sum_{n=0}^{+\infty}\frac{(a\gamma)^n}{(n + 1)!}, \tag{56}$$

which is an increasing function of γ. We can thus deduce that pure phase modulation will decorrelate an input Gaussian field.

5. CONCLUSION

In this paper we have proposed a general formalism for the determination of the spatial correlation of the optical field that is due to a nonlinear transformation of the input signal with SLM's. We have developed this formalism for the case of monotonic phase variation as a function of input voltage.

We have illustrated the application of this method to colored Gaussian noise. In particular, we have demonstrated that linear coupled amplitude phase modulation decorrelates the input noise and decreases the signal-to-noise ratio. We have also shown that the noise is decorrelated with pure phase coding.

The transformation of input noise that is due to EASLM has not been a subject of precise investigations, in particular for the simulation and the characterization of optical signal processing architectures. However, its consequences can be as important as those of an inappropriate signal processing algorithm. The present proposed method should permit progress in this direction. Further generalization of this research can be analyzed in the future. In particular, it should be possible to introduce the effect of nonuniformity (K dependent on x) with this approach.

APPENDIX A

Without loss of generality, the nonlinear relation between the input signal $f(x)$ sent to the EASLM and the optical field $\Psi(x)$ after passage through the EASLM can be written as

$$\Psi(x) = M[f(x)], \tag{A1}$$

with

$$M(y) = D(y)\exp[iB(y)]. \tag{A2}$$

With physical systems one expects that the phase and the modulus of $M(y)$ are continuous functions of y. So, amplitude and phase are continuous functions on a bounded interval. Thus they can be approximated by polynomials with an arbitrary precision.[14] We will thus write

$$D(y) = \sum_{n=0}^{p} d_n y^n, \qquad B(y) = \sum_{n=0}^{q} b_n y^n. \tag{A3}$$

If $B(y)$ is a monotonic function of y [i.e., the phase of the modulation of the optical field is a strictly increasing or decreasing function of the input driving signal $f(x)$], one can consider the new variable defined by

$$B(y) = Kz. \tag{A4}$$

In that case $B(y)$ is a bijective function of y. It is thus always possible to apply to the input signal $f(x)$ an input lookup table such that the input driving signal $V(x)$ at pixel x satisfies

$$B[V(x)] = Kf(x). \tag{A5}$$

The input modulation is thus given by

$$M[f(x)] = A[f(x)]\exp[iKf(x)], \tag{A6}$$

with

$$A[f(x)] = D\{B^{-1}[f(x)]\}. \tag{A7}$$

We will consider an approximation of $A(z)$ with a polynomial:

$$A[f(x)] = \sum_{n=0}^{p} a_n f^n(x). \tag{A8}$$

In the present paper we concentrate on this model. In that case the optical field $\Psi(x)$ is still given by Eq. (1), with

$$M(z) = \sum_{n=0}^{p} a_n z^n \exp(iKz). \tag{A9}$$

APPENDIX B

One has

$$\mathcal{A}_1\mathcal{A}_2^* = \frac{\partial}{\partial K_1}\frac{\partial}{\partial K_2}. \tag{B1}$$

It is easier to determine each term separately. The differential operator $\mathcal{A}_1\mathcal{A}_2^*$ must be applied to

$$\exp[iK_1 f(x) - iK_2 f(y) - {}^1\!/\!_2\Gamma_{x,x}K_1^2 - {}^1\!/\!_2\Gamma_{y,y}K_2^2 + \Gamma_{x,y}K_1K_2]. \tag{B2}$$

One thus has the formal rule

$$\frac{\partial}{\partial K_1} \rightarrow if(x) - \Gamma_{x,x}K_1 + \Gamma_{x,y}K_2, \tag{B3}$$

$$\frac{\partial}{\partial K_1}\frac{\partial}{\partial K_2} \rightarrow [if(x) - \Gamma_{x,x}K_1 + \Gamma_{x,y}K_2] \times [-if(y) - \Gamma_{y,y}K_2 + \Gamma_{y,x}K_1] + \Gamma_{x,y}. \tag{B4}$$

We are now able to determine that

$$\langle\Psi(x)^*\Psi(y)\rangle = \{\Gamma_{x,y} + [if(x) - \Gamma_{x,x}K + \Gamma_{x,y}K] \times [-if(y) - \Gamma_{y,y}K + \Gamma_{y,x}K]\}\exp\{iK[f(x) - f(y)]\} \times \exp\{iK[-{}^1\!/\!_2K^2(\Gamma_{x,x} + \Gamma_{y,y}) + \Gamma_{x,y}K^2]\}. \tag{B5}$$

We have seen that

$$\langle\Psi(x)\rangle = [f(x) + i\Gamma_{x,x}K]\exp[iKf(x) - {}^1\!/\!_2\Gamma_{x,x}K^2]. \tag{B6}$$

Thus

$$\langle\Psi(y)\rangle^*\langle\Psi(x)\rangle = [f(x) + i\Gamma_{x,x}K][f(y) - i\Gamma_{y,y}K] \times \exp\{iK[f(x) - f(y)]\} - {}^1\!/\!_2K^2(\Gamma_{x,x} + \Gamma_{y,y}). \tag{B7}$$

Since

$$C_{\Psi,\Psi}(x, y) = \langle\Psi(y)^*\Psi(x)\rangle - \langle\Psi(y)\rangle^*\langle\Psi(x)\rangle,$$

one thus obtains

$$C_{\Psi,\Psi}(x, y) = [E(x, y) - F(x, y)]L(x, y), \tag{B8}$$

with

$$E(x, y) = \{[if(x) - \Gamma_{x,x}K + \Gamma_{x,y}K] \times [-if(y) - \Gamma_{y,y}K + \Gamma_{y,x}K] + \Gamma_{y,x}\}\exp(\Gamma_{x,y}K^2), \tag{B9}$$

$$F(x, y) = [f(x) + i\Gamma_{x,x}K][f(y) - i\Gamma_{y,y}K], \tag{B10}$$

$$L(x, y) = \exp\{iK[f(x) - f(y)] - {}^1\!/\!_2K^2(\Gamma_{x,x} + \Gamma_{y,y})\}. \tag{B11}$$

ACKNOWLEDGMENTS

The authors acknowledge fruitful discussions with B. Granier and support in this research from J.-P. Huignard.

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