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Coupling characteristics of localized phonons in photonic crystal fibers

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Photonic crystal fibers are candidates for enhanced elasto-optical interactions. The acoustic properties of such structures are investigated theoretically by the finite-difference time-domain method, and a full acoustic band gap is found. The transmission spectrum of acoustic waves through a photonic crystal fiber containing one or two coupled cavities is found to be very sensitive to the geometrical and physical parameters of the structure, as well as to the relative position of the two defects. © *2003 American Institute of Physics.* [DOI: 10.1063/1.1627946]

Acoustic band-gap materials,[1–5] also called phononic crystals, are composite elastic media, constituted of two-dimensional (2D) or three-dimensional periodic repetitions of different solids or fluids, that exhibit stop bands in the transmission spectrum of elastic waves. The location and width of acoustic band gaps result from a large contrast in the value of the elastic constants and/or mass density of the constitutive materials. These materials are receiving increasing attention as potential candidates for the design of elastic-acoustic wave guides[6,7] or filters. When a defect or a cavity is introduced in the otherwise perfect crystal, one can obtain localized modes associated with the defect in the band gap of the phononic crystal.

In a recent publication,[8] Russell and collaborators reported on the realization of a dual-core square-lattice photonic crystal fiber (PCF) presenting acoustic defect modes. Their results show a coupling between the two acoustic defect modes and provide a basis for developing acousto-optical devices based on PCFs. These structures, which are a type of optical fiber waveguide, are all-silica fibers with a two-dimensional array of microscopic air holes running along their length. It is possible to imagine several types of central defects in which light can be guided, e.g., one solid core or a dual solid core, etc. These structures permit the design of optical fibers with formidably guiding properties.[9,10] In the acousto-optical experiment of Marin *et al.*,[8] the optical wave propagating along the core of the PCF couples with phonon defect modes of the core. The acoustic modes are excited by attaching a piezoelectric transducer to one side of the PCF preform. Indeed, the propagation of an acoustic wave in an elastic medium modifies the refractive index through the elasto-optic effect and can, for instance, induce a phase or a polarization change in the optical signal.

The object of this article is to theoretically study the phononic properties of PCFs with a core constituted by one or two coupled cavities made of silica (this means the absence of one or two air holes in the core). We present a comprehensive analysis of the different localized phonon modes that can occur in such a PCF and discuss the sensitivity of the results with respect to the geometrical and physical parameters of the system. This is achieved by using the finite-difference time-domain (FDTD) method.[11] The FDTD method solves the elastic wave equation by discretizing time and space and replacing derivatives by finite differences. The properties of solid/solid and fluid/solid acoustic band gaps calculated with this method were shown to compare very well with experimental measurements.[12] Our calculations of the transmission coefficient are performed for 2D phononic crystals of square geometry composed of air holes in a silica matrix, as depicted in Fig. 1(a). We limit the model to elastic displacements, velocities, and stress fields in the XY plane perpendicular to the cylindrical air inclusions. The phononic crystal is repeated periodically along the X axis, whereas its length along the Y direction is taken to be 7 periods of the elementary cell. Absorbing Mur's boundary conditions are imposed at the free ends of the computation domain along the Y direction.

The physical parameters characterizing the acoustic properties of the constitutive materials are the longitudinal, c_ℓ, and transverse (only for silica), c_t, velocities of sound as well as the mass density ρ. The values $c_\ell = 5968$ m/s, $c_t = 3760$ m/s, and $\rho = 2200\ \mathrm{kg\,m^{-3}}$ for silica, and $c_\ell = 349$ m/s and $\rho = 1\ \mathrm{kg\,m^{-3}}$ for air are used. The geometrical parameters are the lattice period a, and the radius of the

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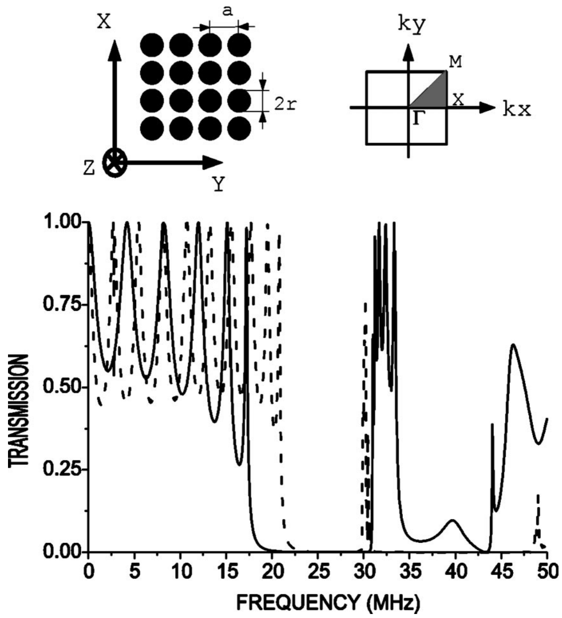

FIG. 1. (a) 2D cross section of the photonic fiber composed of a square array of air cylinders in silica. The irreducible Brillouin zone of the square lattice is also presented. (b) Transmission spectra computed along the ΓX (solid lines) and ΓM (dashed lines) directions of the irreducible Brillouin zone. The lattice parameter and the air cylinder radius are $a=85\ \mu$m and $r=36\ \mu$m, respectively.

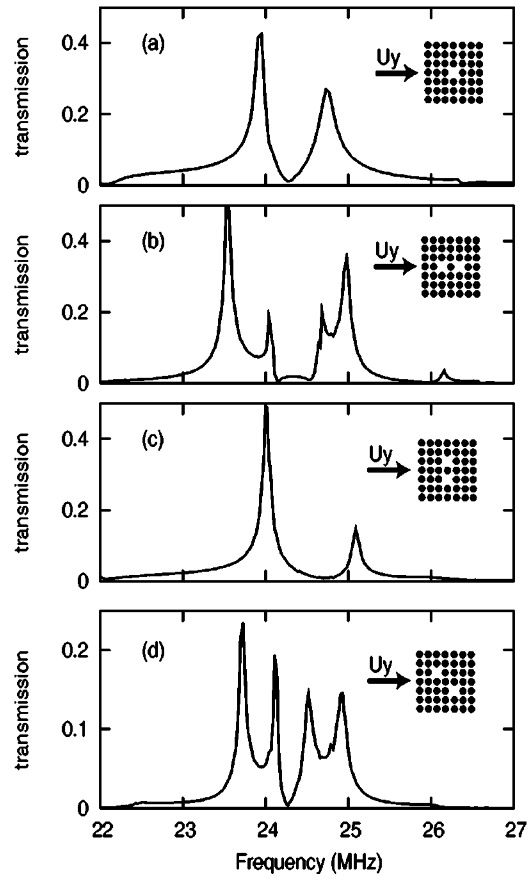

FIG. 2. Transmission vs frequency for interacting defects. (a) One cylinder of air have been filled as shown in the inset. The direction of the incident wave U_y is also indicated in the inset. (b) Two filled cylinders oriented parallel to the incident wave are separated by one air cylinder. (c) Same as in (b), but the orientation of the coupled defects is perpendicular to the incident wave. (d) The coupled defects are oriented at 45° of the incident wave.

inclusion, r. The filling fraction of the composite medium is defined as the ratio $f=\pi r^2/a^2$. Unless stated otherwise, we assume $a=85\ \mu$m and $r=36\ \mu$m. This insures that the phononic crystal constituted by the PCF displays a large acoustic band gap extending from 22 MHz to 29 MHz. This is sketched in Fig. 1, where the transmission spectrum of the phononic crystal is represented as a function of the frequency and is calculated along two high symmetry directions of the Brillouin zone ΓX and ΓM. Let us notice that, in our calculation, the value of $r=36\ \mu$m used for the radius is slightly higher than the one in the experiment of Ref. 8, where $r=34\ \mu$m. This is to insure that defect modes fall in the middle of the complete band gap.

We investigate the defect modes associated with one or two silica cavities introduced in the perfect periodic structure. The defects are created by removing air cylinders, i.e., they are filled with the same silica medium as the matrix. We examine the interaction between a pair of defects as a function of their orientation and separation.

Figure 2 displays several transmission spectra in case the crystal contains one defect or two interacting defects with different orientations. The calculation is performed by using a superperiod along the X direction and a finite number of elementary cells along the Y direction. When one defect is introduced into an otherwise perfect periodic structure, the transmission spectrum [Fig. 2(a)] displays two localized states within the stop bands that appear at 23.95 and 24.73 MHz, respectively. These cavity modes are very sensitive to the radius r of the silica cylinders. A slight modification of r can drastically change the frequencies of the peaks in the transmission spectrum. The most interesting situation, corresponding to two peaks in the middle of the gap, is obtained when r is in the range from 34 μm to 37 μm, which corresponds to a filling fraction f between 0.50 and 0.6. Moreover, it is interesting to notice that the frequency of the lower cavity mode is more sensitive to the value of the longitudinal velocity of sound c_ℓ in silica than to the value of the transverse velocity c_ℓ. The converse behavior is true for the higher cavity mode. Indeed, we show in Fig. 3 the frequency shift of both cavity modes when c_ℓ and c_t are slightly changed from the values used for plotting Fig. 2(a). It may even be possible to obtain a crossing of these modes when the velocities are changed by more than a few percent. This sensitivity can have a great practical impact, since the material constants for silica are only known with a finite precision and can be modified by changing the chemical composition.

Since the cavity possesses two perpendicular mirror planes perpendicular to the X and the Y axes, its eigenmodes can be labeled according to their symmetry with respect to these axes. Nevertheless, in the FDTD calculation, the cavity modes are excited by means of an incident probe wave of longitudinal polarization, i.e., along the Y axis. Then only the modes that are symmetrical with respect to the Y axis can be excited. We report in Fig. 4 maps of the eigenvectors for both cavity modes; theses maps are obtained by considering

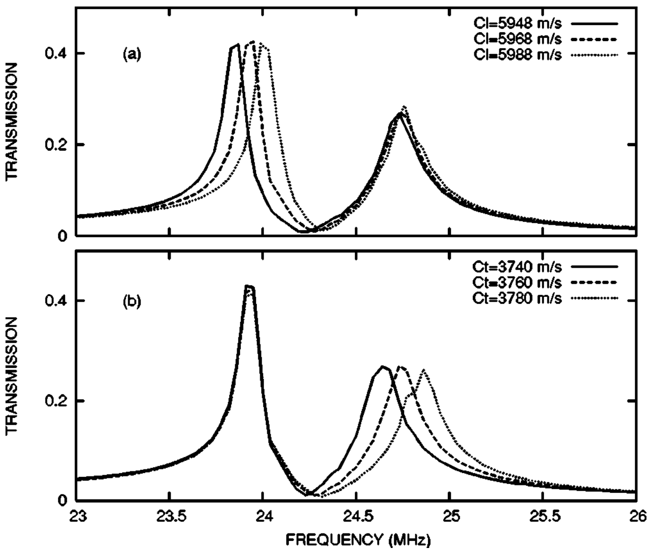

FIG. 3. Same as in Fig. 2(a) for different values of the velocities of sound in silica. In (a) and (b), the longitudinal and transverse velocities of sound are respectively modified.

an incident harmonic probing signal at the frequencies of the peaks in the transmission spectrum of Fig. 2(a). From these maps, one can notice that the eigenvectors are symmetrical with respect to both the X and the Y axes.

Figure 2(b) displays the transmission spectrum for two adjacent cavities along the Y direction, i.e., the direction of propagation. The coupling between the cavities results in the splitting of each of the modes associated with a single cavity. These modes are still symmetrical with respect to the Y axis. However, each pair of modes is, respectively, antisymmetrical or symmetrical with respect to the mirror plane that separates the two cavities. This is confirmed by inspecting the eigenvectors (although they are not shown in Fig. 2).

We have also investigated the effect of changing the separation between the cavities, and therefore their interaction, on the splitting of the modes. We have found that the effect of the coupling between the two defects can be observed as far as the cavities are separated by less than three cylinders. For further separation, the splitting almost disappears and one recovers the modes of a single cavity.

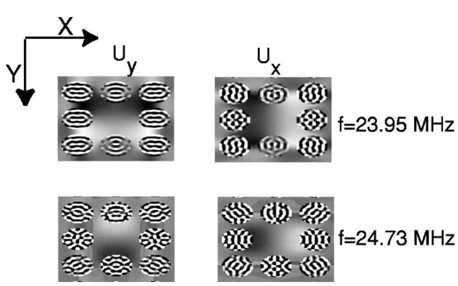

FIG. 4. Maps of the eigenvectors of the cavity modes obtained in Fig. 2(a).

The transmission spectrum of Fig. 2(c) corresponds to the case of two coupled cavities oriented perpendicular to the incident signal, i.e., along the X axis. In this geometry, the modes of the coupled cavities are the same as in the previous case and their symmetries are obtained by interchanging the X and the Y axes. However, due to the longitudinal polarization of the incident probe, only two of these modes can be excited and appear in the transmission spectrum. They correspond to the modes that are symmetrical with respect to the mirror plane of the coupled cavity along Y, while the two other modes, that are antisymmetrical with respect to this axis, cannot be excited by the incident probe wave.

The transmission spectrum of Fig. 2(d) is associated with two adjacent defects that are at 45° of the X and Y axes. Here, again, one can observe a splitting of the individual modes of a cavity, but the interaction between the defects is different from the two previous cases and results in a different splitting of the modes. One can also notice an overall decrease in the amplitude of the transmitted signal.

In summary, it has been verified that the PCF structure studied in this work possesses a full acoustic band gap. The FDTD calculation of the transmission spectrum of acoustic waves through a PCF containing one or two coupled cavities emphasizes the sensitivity of the results with respect to the geometrical and physical parameters of the structure, as well as to the relative position of the two defects. The understanding of the behavior of these localized modes is important to perform and interpret acousto-optical experiments in PCF structures.

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