Guiding and bending of acoustic waves in highly confined phononic crystal waveguides

A. Khelif,[a] A. Choujaa, and S. Benchabane
Institut FEMTO-ST, Département LPMO, CNRS UMR 6174, Université de Franche-Comté, 32 Avenue de l'Observatoire, 25044 Besançon Cedex, France

B. Djafari-Rouhani
Laboratoire de Dynamique et Structures des Matériaux Moléculaires, CNRS UMR 8024, Université de Lille I, 59655 Villeneuve d'ascq Cedex, France

V. Laude
Institut FEMTO-ST, Départment LPMO, CNRS UMR 6174, Université de Franche-Comté, 32 Avenue de l'Observatoire, 25044 Besançon Cedex, France

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We demonstrate experimentally the guiding and the bending of acoustic waves in highly confined waveguides formed by removing rods from a periodic two-dimensional lattice of steel cylinders immersed in water. Full transmission is observed for a one-period-wide straight waveguide within the full band gap of the perfect phononic crystal. However, when the waveguide width is doubled, destructive interference causes the transmission to vanish in the center of the passband. Waveguiding over a wide frequency range is obtained for a one-period-wide waveguide with two sharp 90° bends. Finite-difference time-domain computations are found to be in good agreement with the measurements. © *2004 American Institute of Physics.* [DOI: 10.1063/1.1757642]

The study of acoustic and elastic wave propagation in periodic band-gap materials, known as phononic crystals, has been receiving growing interest in recent years. These composite media typically exhibit stop bands in their transmission spectra for which the propagation of waves is strictly forbidden in all directions.[1–8] This makes these systems potential candidates for the design of elastic or acoustic waveguides or filters. The location and width of acoustic band gaps result from a large contrast in the value of the elastic constants and/or the mass density of the constitutive materials. Therefore, there is a great deal of interest in developing phononic crystal-based waveguides where one can confine and efficiently guide acoustic waves around sharp corners, which is not feasible with classical waveguides. Guiding acoustic waves without losses in straight waveguides using two-dimensional phononic crystals was studied theoretically by several groups.[9–11] However, in contrast to photonic crystals, less attention has been devoted to experimental demonstration,[6,12,13] especially for bent waveguides.[6,12]

The aim of this letter is to demonstrate experimentally that the guiding and the bending of acoustic waves can be achieved in a tight space in highly confined waveguides constructed by removing rods from a perfect two-dimensional phononic crystal. The most significant result is the experimental demonstration of wave transmission through a bent waveguide over a large frequency range, in accordance with our theoretical predictions. The specific arrangements considered experimentally are depicted in Fig. 1. The ultrasonic crystal is composed of a two-dimensional periodic array of an acoustic scattering material (steel) immersed in water. The choice of steel and water as the composite materials is based on the strong contrast in their densities and elastic constants. The square lattice constant is 3 mm and the scatterers diameter is 2.5 mm, resulting in a filling fraction of 55%. For this arrangement, we demonstrated experimentally[13] that there is a full band gap extending between 250 and 325 kHz in which acoustic waves are not allowed to propagate in the crystal. The experimental results were found to be in good agreement with a theory based on a finite difference time domain (FDTD) method.[7–11,14] The experimental setup is based on the well-known ultrasonic immersion transmission technique and was described in Ref. 13. It is only briefly summarized here. A couple of wide-band-width transmitter/receiver acoustic transducers (Panametrics immersion transducers type Videoscan V301) are used. A pulser/receiver generator (Panametrics model 5800) produces a short duration pulse which is applied to the source transducer launching the probing longitudinal waves. The signal detected by the receiving transducer is acquired by the pulser/receiver, postamplified, and then digitized by a digital sampling oscilloscope. To reduce random errors, 500 measurements are averaged before a fast Fourier transform is performed to obtain the transmission spectrum. The system is first calibrated with no sample present; a reference signal is digitized and its spectrum is

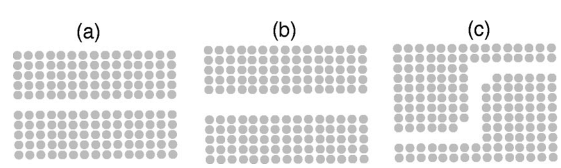

FIG. 1. Cross sections of waveguides formed by removing rods from an ultrasonic crystal made of steel rods immersed in water. (a) A one-period-wide straight waveguide (W1). (b) A two-period-wide straight waveguide (W2). (c) A one-period-wide bended waveguide.

[a]Electronic mail: abdelkrim.khelif@lpmo.edu

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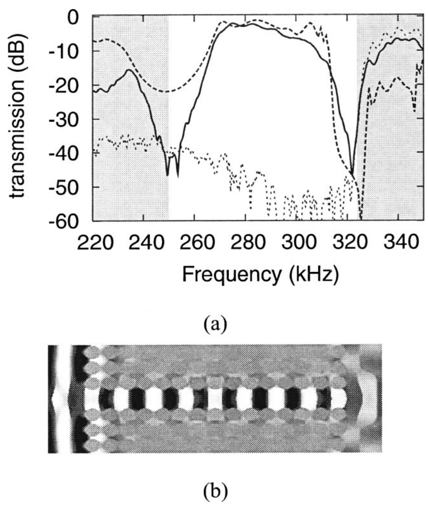

FIG. 2. (a) Experimental (solid line) and calculated (dashed line) transmission power spectra along waveguide W1 of Fig. 1(a). The experimental transmission of the perfect crystal in the Γ*X* direction (thin solid line) is shown for comparison. The gray regions delimit the full band gap for the perfect crystal. (b) Calculated longitudinal displacements averaged over one period at the frequency $f=287$ kHz. Gray levels are representative of the amplitude and range from black for negative to white for positive values.

used to normalize the subsequent transmission spectra. We first consider a straight waveguide, W1, created by removing exactly one row of cylinders along the propagation direction (*X* axis), as shown in Fig. 1(a). The length of the W1 is 15 periods or 45 mm and its width, defined by the distance between neighboring cylinders on both sides of the guide, is 3.5 mm. The measured transmission is displayed in Fig. 2(a). Throughout this letter, transmission values are normalized with respect to the entrance area of the waveguide. We observed full transmission of acoustic waves for certain frequencies within the phononic crystal stop-band. Full transmission within the waveguiding band is a strong though indirect indication that the wave is well confined within the waveguide and is guided with weak losses. The guiding band starts at 260 kHz and ends at 315 kHz, covering a large part of the stop band. For comparison, Fig. 2(a) also displays the transmission spectrum for the perfect crystal along the *X* axis. The FDTD computation is seen to agree correctly with the measurements. To obtain a numerical confirmation of the waveguiding properties, the FDTD computation was used to simulate a monochromatic source at frequency $f=287$ kHz. The computed longitudinal displacements are displayed in Fig. 2(b). It is clearly seen that the part of the wave that is incident on the phononic crystal is fully reflected backwards, but that within the waveguide section the wave propagates without attenuation. At the exit of the waveguide, the wave is strongly diffracted since the output aperture is smaller than a wavelength in water ($\lambda=5.2$ mm at 287 kHz).

The effect of the width of the waveguide was evaluated by performing the same measurements with waveguide W2 depicted in Fig. 1(b). Exactly two cylinders' rows are now removed and the waveguide width is 6.5 mm. The measured transmission and the corresponding FDTD computation are shown in Fig. 3(a). Two distinct waveguiding bands are now observed within the stop band. In particular, a strong attenuation is observed around 285 kHz. To explain this phenomenon, the FDTD computation is again used to simulate a

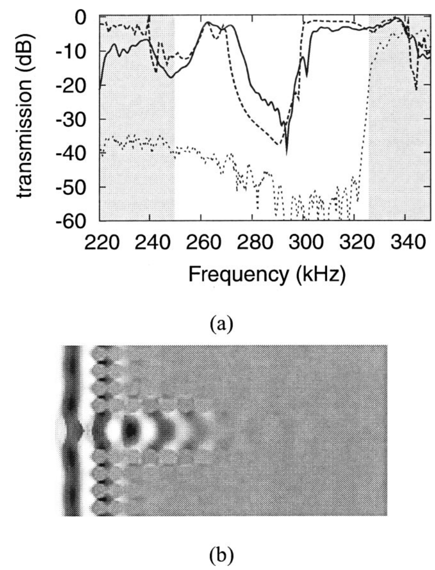

FIG. 3. (a) Same as Fig. 2(a) but for waveguide W2 of Fig. 1(b). (b) Same as Fig. 2(b) at the frequency $f=287$ kHz.

monochromatic source at 287 kHz. The computed longitudinal displacements are displayed in Fig. 3(b). At that frequency, it is apparent that destructive interferences occur, resulting in the creation of a stop band in the waveguide band structure, as explained latter. It is worthwhile noting that such a phenomenon is not expected in classical waveguides and is here the result of the periodicity of the crystal structure. In order to understand the differences between waveguides W1 and W2, it is useful to obtain the one-dimensional band structures for the waveguide modes as reported in Fig. 4. They are obtained through the FDTD calculation by defining a super-cell of seven periods in the *Y* direction, i.e., the waveguides are repeated periodically every seven cells. It is expected that within the full band gap the

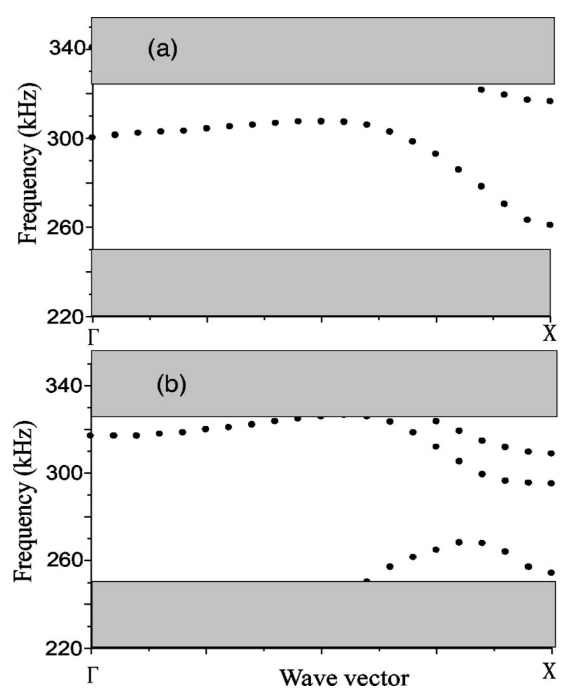

FIG. 4. Super-cell calculation of the dispersion relation for (a) waveguide W1 and (b) waveguide W2.

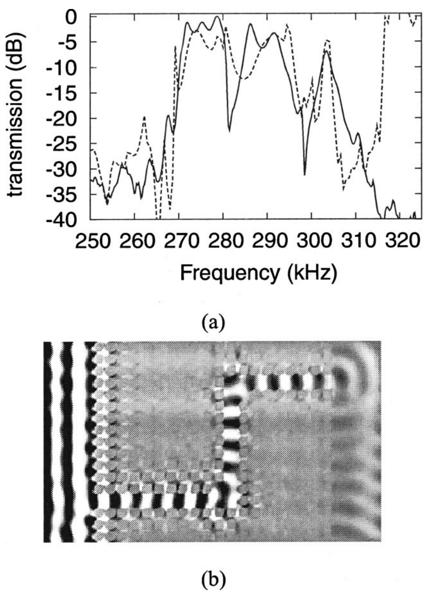

FIG. 5. (a) Same as Fig. 2(a) but for the one-period-wide bended waveguide of Fig. 1(c). (b) Calculated acoustic pressure averaged over one period at the frequency $f=275$ kHz.

periodized waveguides are not able to interact due to the strong evanescence of waves that would couple them. In Fig. 4, the full band gap for the perfect phononic crystal is delimited by two gray regions. The dispersion curves that are found inside it are those of localized modes associated with the straight waveguide. The number and the dispersion of localized modes can be understood, and qualitatively explained, by considering the propagation of acoustic waves in a classical linear guide with rigid boundary conditions, for which the dispersion relation can be derived analytically.[15] The main difference between phononic crystal waveguides and classical ones is that they are delimited by rough rather than planar walls. The roughness of the walls has the periodicity a of the steel cylinders along the X direction. Therefore, the dispersion curves of a classical guide should essentially be folded back into the reduced Brillouin zone of dimension π/a. The numerous localized branches inside a full band gap are the result of this folding, and the interaction of branches that cross each other induces band gaps in the transmission of the waveguide. In the case of waveguide W1, only a small waveguide band gap is expected from Fig. 4(a), near the upper end of the full band gap. However, in the case of W2, a large waveguide band gap is created in the center of the full band gap, as depicted in Fig. 4(b). This observation is consistent with the experimental and computed results of Fig. 3(b).

Finally, we tested the bending of acoustic waves through two sharp corners (90°) in the W1 structure shown in Fig. 1(c). As shown in Fig. 5(a), we observe a main waveguiding band extending from 270 to 310 kHz. This spectrum is altered by two transmission drops around 281 and 299 kHz. As a result, the whole waveguiding band for the sharply bended structure covers approximately 70% of the full band gap. Figure 5(b) is a numerical illustration of the acoustic pressure within the waveguide at 275 kHz. The acoustic pressure, $p=\nabla\cdot\mathbf{u}$, includes both the displacements u_x and u_y. The incident wave propagates along the first straight part of the waveguide, successfully couples to the second part with the same shape, although the latter is perpendicular to the former, and again couples to the third part of the waveguide.

In conclusion, we have investigated experimentally and theoretically the guiding and tight bending of acoustic waves in a binary two-dimensional phononic crystal constituted by a square array of circular parallel steel cylinders in water. We have demonstrated that a fully confined waveguide can be constructed by removing a single row in the phononic crystal. Full transmission of the acoustic wave was obtained through a straight waveguide and a waveguide with two sharp bends. Moreover, we have observed that this full transmission can be reduced when the width of the waveguide is doubled, as a result of the creation of a stop band for waveguide modes within the full band gap.

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