Complete band gaps and deaf bands of triangular and honeycomb water-steel phononic crystals

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Phononic crystals with triangular and honeycomb lattices are investigated experimentally and theoretically. They are composed of arrays of steel cylinders immersed in water. The measured transmission spectra reveal the existence of complete band gaps but also of deaf bands. Band gaps and deaf bands are identified by comparing band structure computations, obtained by a periodic-boundary finite element method, with transmission simulations, obtained using the finite difference time domain method. The appearance of flat bands and the polarization of the associated eigenmodes is also discussed. Triangular and honeycomb phononic crystals with equal cylinder diameter and smallest spacing are compared. As previously obtained with air-solid phononic crystals, it is found that the first complete band gap opens for the honeycomb lattice but not for the triangular lattice, thanks to symmetry reduction. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2472650]

INTRODUCTION

The study of acoustic and elastic wave propagation in phononic crystals has been receiving great interest recently. Phononic crystals are inhomogeneous elastic media composed of one-, two-, or three-dimensional periodic arrays of inclusions embedded in a matrix. Several classes of phononic crystals exist that differ mainly by the physical nature of the inclusion and of the matrix. The solid/solid, fluid/fluid, and the mixed solid/fluid composite systems have attracted attention. These composite media can exhibit stop bands in their transmission spectra for which the propagation of waves is strictly forbidden in all directions.[1–8] Such complete acoustic band gaps result from a large contrast in the value of the elastic constants and/or the mass density of the constitutive materials. By adding a line defect in the periodic media, it is possible to create highly confined guides for acoustic and elastic waves.[9–11] Such phononic waveguides can confine and efficiently guide acoustic waves around sharp corners.

One of the goals of the study of phononic crystals is the search for material combinations or structures producing large gaps at some desired frequency range. It has been shown that complete band gaps can be enlarged by decreasing the lattice symmetry.[5,12–14] We consider in this work the triangular lattice with one or two scatterers in the unit cell, the latter case being referred to as the honeycomb lattice, and compare these two structures. Specifically, we investigate experimentally the transmission of ultrasonic acoustic waves through two-dimensional phononic crystals consisting of steel cylinders immersed in water, and compare it to the acoustic band structure obtained by numerical analysis. The choice of steel and water as the composite materials is based on the strong contrast in their densities and elastic constants and has proven useful for the fabrication of square-lattice phononic crystals.[13,15] It is observed that, for both the triangular and the honeycomb case, deaf bands appear in addition to band gaps, in accordance with the results of Sanchez-Perez *et al.* for air/solid triangular sonic crystals.[5] Deaf bands are acoustic branches of modes that cannot be excited depending on the symmetry of the mode with respect to the source, hence they do not transport acoustic energy through the crystal. In the usual case that a plane wave is normally incident on the phononic crystal, a mode that is antisymmetric with respect to the propagation direction will be deaf. We show that band gaps and deaf bands can be identified by comparing band structure computations for the infinite structure, obtained by a periodic-boundary finite element method, with transmission simulations for the finite structure, obtained using the finite difference time domain method.

METHODS

Figure 1 displays the experimental setup used in our experiments. Triangular-lattice phononic crystals were constructed using steel cylinders with a diameter $d = 1.2$ mm and a length of 150 mm. The nearest distance between the cen-

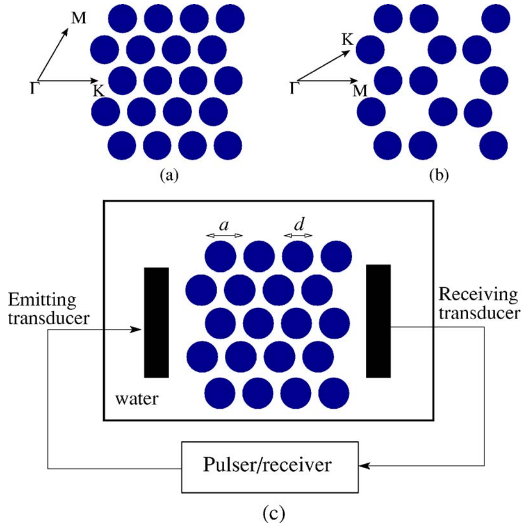

FIG. 1. (Color online) Definition of the propagation directions for (a) triangular and (b) honeycomb phononic crystals, and (c) experimental setup used to measure transmission spectra using acoustic transducers immersed in water.

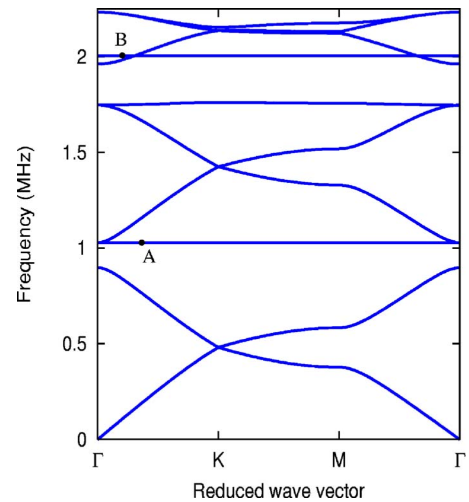

FIG. 2. (Color online) Band structure of the infinite triangular-lattice phononic crystal composed of steel cylinders in water, plotted along the $\Gamma-K-M-\Gamma$ path of the first irreducible Brillouin zone.

ters of two steel cylinders is $a=1.5$ mm. Alignment of the cylinders is obtained by using two supporting plates in which a periodic array of holes has been machined. The honeycomb lattice is simply formed from the triangular one by removing a steel cylinder of the central hexagonal unit cell. Because of the isotropy of elastic wave propagation in the materials considered, complete band gaps can in principle be identified experimentally from the transmission spectrum measured along the two highest symmetry directions. These two directions correspond to the ΓK and the ΓM directions of the first irreducible Brillouin zone, respectively. The experimental arrangement for measuring the transmission along these two directions is depicted in Fig. 1. The measurement procedure is based on the well-known ultrasonic immersion transmission technique and was described in Ref. 13. Two sets of transducers are used in order to cover a wide spectrum. Their central frequencies are around 450 and 900 kHz, respectively. By overlapping transmissions, the whole measured spectrum runs from 100 to 1200 kHz.

The finite difference time domain (FDTD) method is used to compute the transmission through the finite-thickness phononic crystal samples, following the procedure described in detail in Ref. 15. The output of the FDTD method can thus be compared directly with transmission measurements. It is worth noting that both the experimental set up and the FDTD computation only measure the transmission of waves that are normally incident on the samples. Therefore, Bragg scattering to diffraction orders larger than zero in the water surrounding the phononic crystals is not taken into account in our analysis. In addition, the band structure of the (infinite) phononic crystals is computed using a finite element method (FEM) with periodic boundary conditions.[16,17] The FEM computation also provides us with the eigenmodes associated with each frequency-wave vector point in the band structure. Thus, it is possible to check whether a given band in the band structure is deaf or not, as explained below. In addition, in the case of flat bands, the localization of elastic energy within the scatterers or the matrix can be checked.

TRIANGULAR PHONONIC CRYSTAL

Before turning to the experimental results, we display in Fig. 2 the complete band structure for the triangular lattice. This plot shows the frequency versus the reduced wave vector along the first irreducible Brillouin zone. We observe the existence of two complete bandgaps inside which neither vibration nor propagation are allowed for all directions. The first and the second complete band gaps extend from 884 to 1029 kHz, and from 1761 to 1963 kHz, respectively. Only the first one is observable in our experiments. We also note the existence of two flat bands. Modes associated with a flat band should have a group velocity equal or close to zero and exhibit strong spatial localization. In practice, such localized modes are often created by inserting a defect in a periodic structure, which constitutes a cavity. However, this is not the case of our purely periodic structure. In order to check the localization of these modes, the eigenmodes associated with the frequency-wave vector points labeled A and B in the band structure of Fig. 2 are displayed in Figs. 3 and 4, respectively. It can be seen from Fig. 3 that the first flat band is

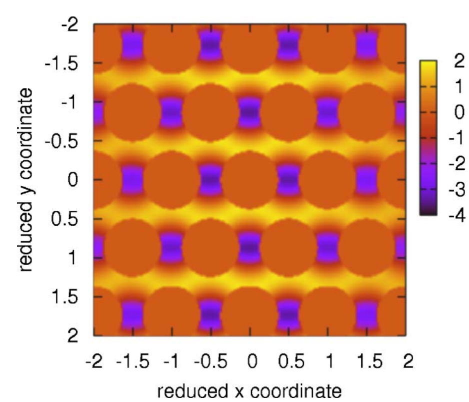

FIG. 3. (Color online) Real part of the pressure field in water for the eigenmode labeled A in the band structure of Fig. 2.

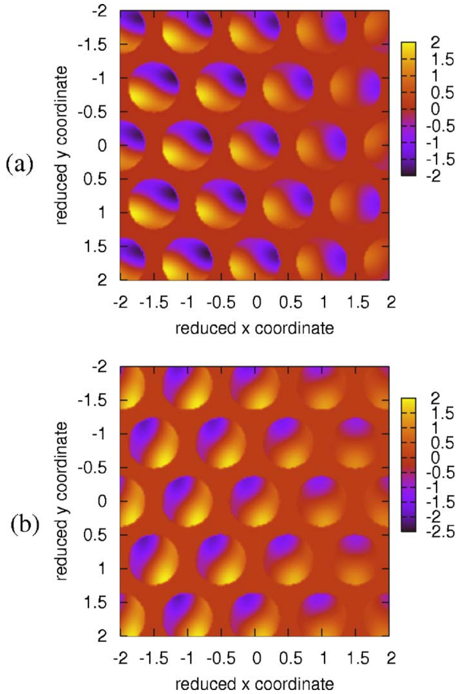

FIG. 4. (Color online) Real part of the (a) x and (b) y displacement field in the steel rods for the eigenmode labeled B in the band structure of Fig. 2.

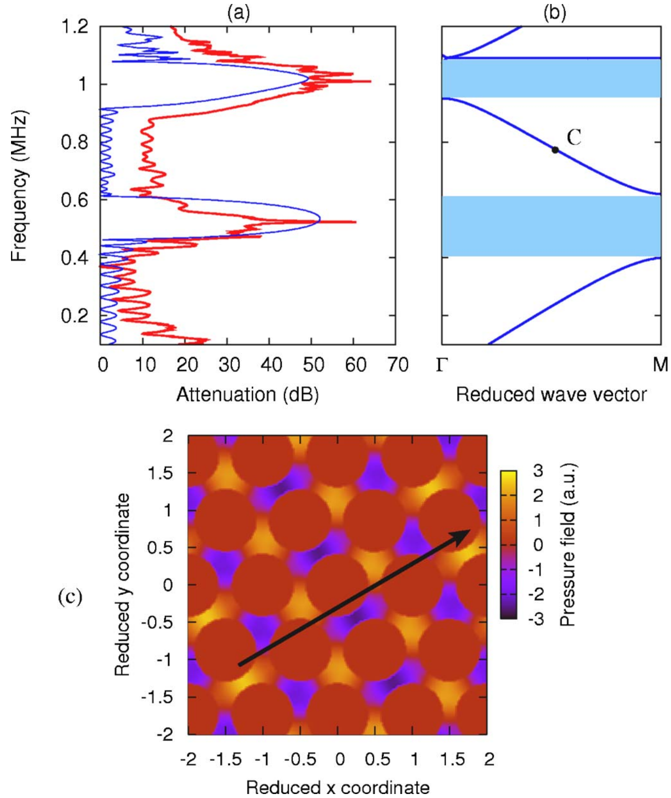

FIG. 5. (Color online) (a) Experimental (thick red solid line) and theoretical (thin blue solid line) transmission through a triangular-lattice phononic crystal of steel cylinders in water, along the ΓM direction. (b) Band structure of the infinite phononic crystal. (c) Real part of the pressure field in water for the eigenmode labeled C in the band structure. The arrow shows the direction of incidence of waves launched by the transducer.

associated with a pressure mode that is localized in the water matrix and that degenerates with the fourth band at the Γ point. Displacements inside the steel rods are zero to numerical errors. However, as illustrated by Fig. 4, the flat band above the second complete band gap is associated with a mode localized inside each individual steel rod. This mode has in-plane elastic displacement in the steel rod, but the pressure field in the water matrix vanishes to numerical errors. The rods do not interact with each other through the water matrix and hence cannot be excited by a wave incident from water. The resonance frequency scales with the inverse of the radius of the rod.

The transmission properties of the triangular-lattice were first evaluated experimentally by performing two transmission measurements through the phononic crystal arranged such that the entrance surface of the phononic crystal is either perpendicular to the ΓM direction or the ΓK direction, as depicted in Figs. 5 and 6, respectively. In Fig. 5, a strong attenuation is observed from 461 to 612 kHz and from 914 to 1078 kHz in the ΓM direction, in good agreement with the FDTD computation. Two band gaps are also clearly apparent in the band structure. The FDTD computation predicts a transmission that is larger than the measured one in between the two band gaps (transmission due to the second acoustic branch) and above the second gap (transmission due to the third and the fourth branches). However, these branches are not found to be deaf with the FEM computation. For instance, Fig. 5(c) displays the pressure field associated with the point labeled C on the second branch of the band structure. This mode is symmetrical with respect to the ΓM propagation direction.

In Fig. 6, a strong attenuation is found to extend from 555 to 1097 kHz in the ΓK direction, both in the measurements and in the FDTD computation. However, the band structure suggests that the band gap is responsible for the attenuation only in the frequency range from 950 to 1097 kHz. In the frequency range 550 to 950 kHz, the second acoustic branch is in principle present but is actually deaf. A similar observation for triangular lattice air-solid sonic crystals was made in Ref. 5 and confirmed in Ref. 18 by using a phase-shift analysis. The pressure field associated with the point labeled D on the second branch of the band structure is displayed in Fig. 6(c). This eigenmode is antisymmetric with respect to the propagation direction. Then a line integral taken along the direction perpendicular to the propagation direction vanishes, and the coupling with the incident plane wave is zero. A similar computation shows that the fourth branch is also deaf. However, the third flat branch is not, and a retransmission at its frequency is observed in the transmission spectrum.

Overlapping the transmission spectra in Figs. 5 and 6, and without consideration of the band structure, it would be tempting to conclude that two complete band gaps exist between 555 and 612 kHz and between 914 and 1078 kHz. However, the former is actually a combination of a band gap in one direction and of a deaf band in the other, and hence is not a true complete band gap. For instance, a defect-based waveguide managed in the triangular-lattice phononic crystal would only operate in the upper band gap, and would be leaky in the first.

The band structures in Figs. 5 and 6 appear to be rather

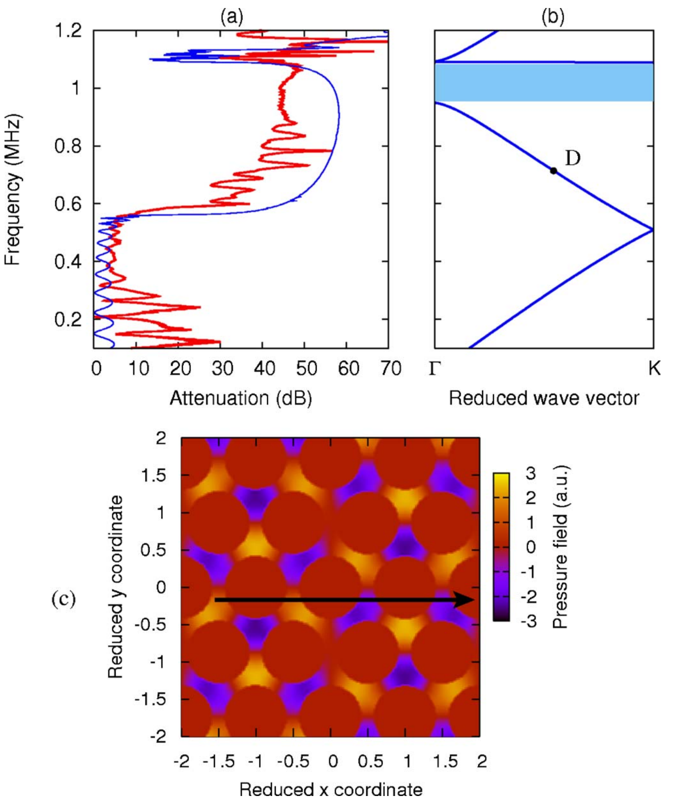

FIG. 6. (Color online) (a) Experimental (thick red solid line) and theoretical (thin blue solid line) transmission through a triangular-lattice phononic crystal of steel cylinders in water, along the ΓK direction. (b) Band structure of the infinite phononic crystal. (c) Real part of the pressure field in water for the eigenmode labeled D in the band structure. The arrow shows the direction of incidence of waves launched by the transducer. This mode belongs to a deaf band and is not excited by a plane-wave transducer.

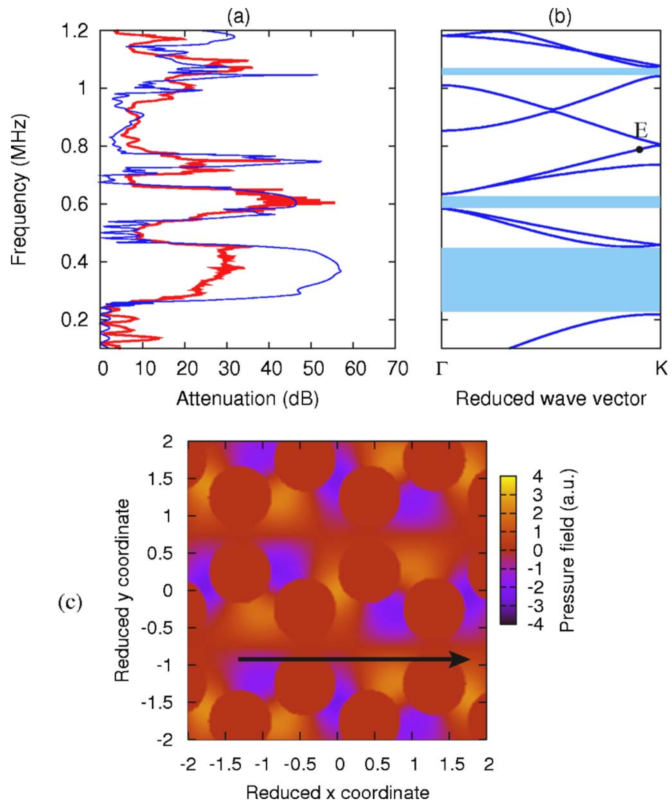

FIG. 7. (Color online) (a) Experimental (thick red solid line) and theoretical (thin blue solid line) transmission through a honeycomb phononic crystal of steel cylinders in water, along the ΓK direction. (b) Band structure of the infinite phononic crystal. (c) Real part of the pressure field in water for the eigenmode labeled E in the band structure. The arrow shows the direction of incidence of waves launched by the transducer. This mode belongs to a deaf band.

similar. One striking difference, however, is that the first branch folding does not open a band gap along the ΓK direction. The first and the second branch yield eigenmodes that are not interacting. As a consequence, the first complete band gap does not open with the triangular lattice.

HONEYCOMB PHONONIC CRYSTAL

The transmission properties of the honeycomb phononic crystal were then evaluated following the same procedure as for the triangular phononic crystal. Figures 7 and 8 display the results for propagation along the ΓK and the ΓM direction, respectively. Again, the combination of the transmission and the band structure gives insight in the band gap properties. For the ΓK direction, Fig. 7, there are three band gaps between the first and the second band, from 258 to 466 kHz, between the third and the fourth band, from 538 to 674 kHz, and between the seventh and the eighth bands, from 1050 to 1100 kHz. In addition, the fifth band is deaf, which causes an attenuation in the transmission between 724 and 776 kHz. The pressure field associated with the point labeled E on the fifth band is displayed in Fig. 7(c). This eigenmode is antisymmetric with respect to the propagation direction. It can be noted that the folding of the fifth to the sixth band does not create a band gap.

For the ΓM direction, Fig. 8, there are four band gaps between the first and the second band, from 244 to 440 kHz, between the third and the fourth band, from 573 to 626 kHz, between the fifth and the sixth band, from 760 to 830 kHz, and between the seventh and the eighth bands, from 980 to 1000 kHz. Again, the fifth band is deaf, which causes a strong attenuation from 660 to 760 kHz. The pressure field associated with the point labeled F on the fifth band is displayed in Fig. 8(c). This eigenmode is antisymmetric with respect to the propagation direction. The ninth band is also deaf. Overlapping the results for the ΓK and the ΓM directions, we find there are two complete band gaps between 258 and 440 kHz, and between 573 kHz and 626 kHz. The widths of these complete band gaps are 182 kHz and 53 kHz, respectively.

DISCUSSION AND CONCLUSION

Comparing the triangular and the honeycomb phononic crystal, we note that the complete band gaps in the honeycomb case appear at lower frequencies than in the triangular case. To be more specific, we consider reduced band gap frequencies by multiplying the frequency with the lattice constant and dividing with the velocity of longitudinal waves in water. The lattice constant in the triangular case is the distance between the centers of two nearest steel cylinders. In the honeycomb case, the lattice constant is the distance between the centers of two nearest hexagonal cells. This means that the lattice constant of the honeycomb case is $\sqrt{3}$

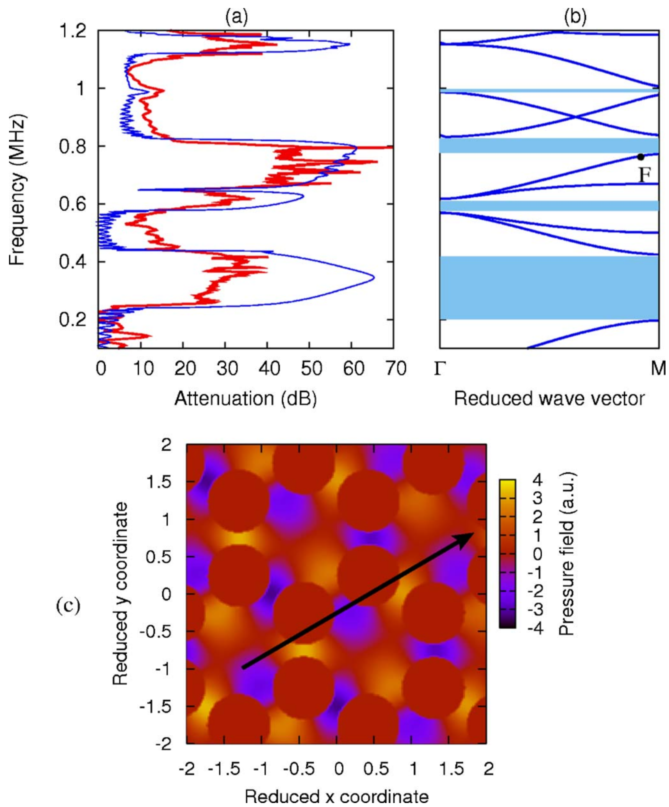

FIG. 8. (Color online) (a) Experimental (thick red solid line) and theoretical (thin blue solid line) transmission through a honeycomb phononic crystal of steel cylinders in water, along the ΓM direction. (b) Band structure of the infinite phononic crystal. (c) Real part of the pressure field in water for the eigenmode labeled F in the band structure. The arrow shows the direction of incidence of waves launched by the transducer. This mode belongs to a deaf band.

times the lattice constant of the triangular case. The complete band gaps of the honeycomb structure show up in the reduced frequency range from 0.45 to 0.77 (fractional bandwidth 52%), and from 1 to 1.09 (fractional bandwidth 9%), while that of the triangular case show up from 0.92 to 1.08 (fractional bandwidth 17%). We have already noted that the first complete band gap in the triangular case (between the first and the second band in Figs. 5 and 6) does not open because the second band is deaf in the ΓK direction. When steel cylinders are removed from the triangular lattice to form the honeycomb lattice, the symmetry reduces and the first complete band gap opens, as mentioned in Ref. 14 in the case of the air/solid composition.

In summary, we have investigated experimentally and theoretically the band gap properties of triangular and honeycomb two-dimensional phononic crystals made of steel cylinders immersed in water. The same geometric parameters have been used to allow for a fair comparison of the two lattices. Using a combination of transmission measurements and band structure calculations, we have identified the lowest complete band gaps in both kind of lattice, but also the appearance of deaf bands that lead to a reduced attenuation in transmission without implying a band gap. As previously obtained with air-steel phononic crystals, it is found that the first complete band gap opens for the honeycomb lattice but not for the triangular lattice thanks to symmetry reduction.

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