Research School

in Number Theory and Applications

Institut de Mathématiques et de Sciences Physiques

Porto-Novo, Bénin

December 6-18, 2021

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Schedule

	8am30-10am30	10am45-12am30	2pm-3pm	3pm -4pm	4pm-5pm
Dec. 7	Opening Session - TE	CM	YRL	YRL	open questions
Dec. 8	CM	TE	YRL	YRL	open questions
Dec. 9	TE	CM	YRL	YRL	YRL
Dec. 10	TE	CM	AFRIMath	AFRIMath	exercices
Dec. 11	CM	TE	YRL	–	–
Dec. 12	–	–	–	–	–
Dec. 13	TE	CM	CA	AQ	YRL
Dec. 14	CM	TE	YRL	YRL	YRL
Dec. 15	TE	CM	YRL	YRL	YRL+LT
Dec. 16	CM	TE	exercices	YRL	YRL
Dec. 17	TE	CM	exercices	exercices	exercices

AQM: Anne Quéguiner Mathieu, University Paris 13, France.
Quadratic forms: from number theory to algebra and geometry.

CA: Cécile Armana, University of Franche-Comté, France.
A short introduction to modular forms.

TE: Tony Ezome, University of Masuku Franceville, Gabon.
On rational points of algebraic curves over number fields.

CM: Christian Maire, University of Franche-Comté, France.
Introduction to class field theory.

LT: Lara Thomas
Danse tes maths.

YRL: Young Researchers Lectures.

AFRIMath: Number Theory Seminar of AFRIMath.

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Young Researchers Lectures

Kouèssi Norbert ADEDJI

On December 7. *On the familly of Diophantine pairs* $\{P_{2k}, 2P_{2k+2}\}$.

Let k be a positive integer such that $k \geq 1$. In this talk, we consider the problem of extension of Diophantine pairs $\{P_{2k}, 2P_{2k+2}\}$. Pell sequence $\{P_k\}_{\geq 0}$ is defined with $P_0 = 0,\ P_1 = 1$ and $P_{k+2} = 2P_{k+1} + P_k$ for $k \geq 0$, while Pell-Lucas sequence $\{Q_k\}_{\geq 0}$ is given by $Q_0 = 2,\ Q_1 = 2$ and the same recurrence relation $Q_{k+2} = 2Q_{k+1} + Q_k$. We prove that if d is a positive integer such that

$$\{P_{2k}, P_{2k+2}, 2P_{2k+2}, d\}$$

is a Diophantine quadruple with $k \geq 1$, then $d = P_{2k+1}Q_{2k+1}Q_{2k+2}$. We deduce that the pair $\{P_{2k}, 2P_{2k+2}\}$ cannot be extended to an irregular Diophantine quadruple. In the proof, we use Baker's theory on linear forms in logarithms of algebraic numbers. This is joint work with Alan Filipin and Alain Togbé.

Johnathan DJELLA LEGNONGO

First Lecture on December 9. *On Weil conjectures.*

Second Lecture on December 16. *Galois extensions of rings.*

Gisèle ETCHIO Martine SOWANOU

On December 15. *Tests de primalité.*

Roslan IBARA

First Lecture on December 9. *On the ideal group of the normal closure of* $\mathbb{Q}(\sqrt[p]{n})$.

Second Lecture on December 14. *Leopoldt's Conjecture.*

Latévie Mohamed LAWSON

on December 15. *Introduction to Hopf algebras.*

Brice MIYAOKA

First Lecture on December 11. *Rational Points on Hyperelliptic Curves.*

Second Lecture on December 15. *Computing rational points on hyperelliptic curves.*

Sarielle PEKA

On December 13. *Efficient Endomorphisms for computing pairings*

Ephraim PONCHO-KOTEY

First Lecture on December 8. *Solving Kirkman School Girls problem Using Field Extension.*

Second Lecture on December 14. *Algebraic surfaces.*

Karim SANKARA

FIRST LECTURE ON DECEMBER 9. *Encoding to algebraic curves and applications to cryptography.*

SECOND LECTURE ON DECEMBER 16. *Aspects algebrico-géométrique des fonctions d'encodages utilisées en cryptographie.*

Safia SEFFAH

ON DECEMBER 8. *k-Pell numbers which are Padovan or Perrin numbers.*

For an integer $k \geq 2$, let $P_n^{(k)}$ be the k-generalized Pell sequence which starts with $0, \ldots, 0, 1$ (k terms) and each term afterwards is given by the linear recurrence

$$P_n^{(k)} = 2P_{n-1}^{(k)} + P_{n-2}^{(k)} + \cdots + P_{n-k}^{(k)}. \quad \text{for all } n \geq 2.$$

The objective of this talk is to find all the k-generalized Pell numbers which are Padovan or Perrin numbers i.e., we solve the Diophantine equation $P_n^{(k)} = P_m$ and $P_n^{(k)} = E_m$ in positive integers n, k, m with $k \geq 2$.

Euloge TCHAMMOU

FIRST LECTURE ON DECEMBER 7. *Fibonacci numbers as sum of same powers of consecutive Fibonacci numbers.*

Introduced in 1202 by the Italian Leonardo Pisano Bigollo $(1180 - 1250)$, the Fibonacci sequence appeared in a subject we now know as population dynamics: For instance, how quickly would a population of rabbits expand under appropriate conditions? The main topic of this presentation is the study of Fibonacci numbers which are sum of same powers of consecutive Fibonacci numbers. For that we give all the solutions of the Diophantine equation $F_n^x + F_{n+1}^x + \cdots + F_{n+k-1}^x = F_m$, where F_i is the i^{th} term of the Fibonacci sequence given by

$$F_0 = 0, \; F_1 = 1 \text{ and } F_n = F_{n-1} + F_{n-2}, \quad \text{for } n \geq 2.$$

Note that earlier in 2011, it has been proved by F. Luca and R. Oyono that a term of the Fibonacci sequence is never a perfect higher power of another term and that a sum of same powers of two consecutive terms cannot be a term apart from the family of identities $F_n^2 + F_{n+1}^2 = F_{2n+1}$, when they studied the Diophantine equation $F_n^s + F_{n+1}^s = F_m$. We gave a nice extension of their result, studying the Diophantine equations $F_n^x + F_{n+1}^x + \cdots + F_{n+k-1}^x = F_m$. We used for that the theory of linear forms in logarithms, the theory of continued fractions, Baker's method of linear forms in logarithms as well as the theorem of *Zeckendorf* representation. Specially, we used three well-known results-one due to Matveev, the second one due to Laurent, Mignotte, and Nesterenko and finally Legendre's criterion on Diophantine approximation. We prove that besides the trivial solutions the only solution of the equation is the one given by $F_3^3 = F_6$.

SECOND LECTURE ON DECEMBER 14. *On some Diophantine equations involving balancing numbers.*

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