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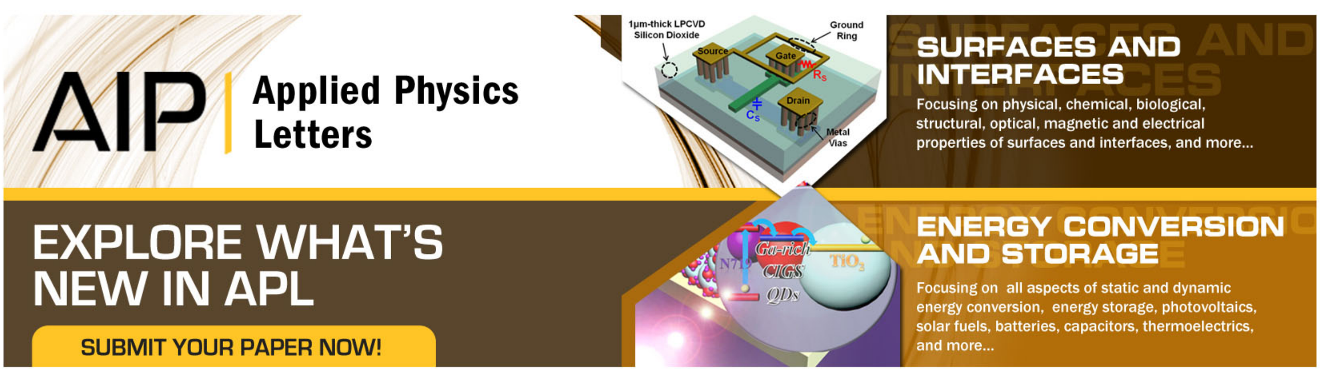

Blazed phononic crystal grating

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It is well known that blazed optical diffraction gratings can significantly increase the diffraction efficiency of plane waves for a selected angle of incidence. We show that by combining blazing with a phononic band gap, diffraction efficiency approaching 100% can be achieved for acoustic waves. We obtain experimentally 98% diffraction efficiency with a two-dimensional phononic crystal of rotated steel rods of square cross-section immersed in water. This result opens the way toward the design of efficient phononic crystal gratings.
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Plane wave diffraction on periodic corrugated surfaces or diffraction gratings appears when the incident wavelength is of the same order as a grating pitch. Blazing a grating means shaping the interior of one groove, or periodic unit-cell, so as to maximize diffraction efficiency to a given order of diffraction. Diffraction efficiency is here understood as the part of the incident energy that goes to a particular diffracted plane wave. Blazing is best understood with the Littrow configuration with sawtooth profile, as depicted in Fig. 1(a). The facet of the grating is made locally orthogonal to the incident light beam and the m-th diffraction order is in auto-collimation, so that the grating law gives

$$2\sin|\theta_i| = |m|\frac{V}{fa}, \qquad (1)$$

where $\theta_m = -\theta_i$, θ_i being the angle of incidence and θ_m being the angle of the m-th order of diffraction, a is the corrugation pitch, V is the wave velocity in the incident medium ($V = 1480$ m/s in water), and f is the frequency. Blazing, however, does not necessarily imply a sawtooth profile. The blazing effect was, for instance, obtained by Wirgin and Deleuil[1] for a grating with rectangular profile, with maximum diffraction efficiency around 98%. In the context of two-dimensional periodic media, the blazing of photonic crystals of holes or rods was studied in theory by Popov *et al.*[2] Computations showed that a two-dimensional photonic crystal of holes or cylinders has blazing properties similar to a classical diffraction grating. In particular, a diffraction efficiency of 97% was observed in numerical simulation for the -1st order. We are unaware, however, of an experimental realization for photonic crystals.

A problem that exists in acoustics, but not for reflective metallic optical diffraction gratings, is that transmission never vanishes for a slab of natural material. As a result, Fabry-perot like resonances can appear, leading to phenomena such as extraordinary acoustic transmission.[3,4] Resonant transmission may thus ruin the operation of reflective acoustic diffraction gratings. In contrast, efficient non-resonant reflection can be obtained for all frequencies lying in a band gap of a phononic crystal (PnC). PnCs are two- or three-dimensional periodic structures that are made of at least two materials with different mechanical properties. They are acoustic analogs of photonic crystals for optical waves.[5] PnC can exhibit complete band gaps, i.e., finite continuous frequency regions where energy propagation is forbidden for all possible wave directions,[6] or conversely where only evanescent waves are allowed.[7] Band-gap ranges and widths mainly depend on the materials employed in the crystal construction, and on the lattice geometry, the size, and the shape of any inclusion.[8,9] In a previous paper,[10] we studied a square-lattice PnC of steel rods in water from the perspective of diffraction gratings. It was demonstrated, in particular, that the angles of propagation of diffraction orders in the surrounding medium are governed by the periodic boundaries between the incident medium and the PnC, in accordance with the grating law of which Eq. (1) is a particular case for the Littrow condition. Diffraction efficiencies, however, are governed by a rich interplay with the Bloch waves of the PnC. In this letter, we study in what manner the diffraction efficiency of PnC can be optimized by combining blazing with a phononic band gap. In particular, we wish to determine if it is possible to get diffraction efficiencies approaching 100%.

A specific blazed two-dimensional phononic crystal of rotated square rods was designed. Previous theoretical studies on the influence of physical rotation were conducted using the plane wave expansion method for fluid-fluid, solid-solid (isotropic case) for PnC of rectangular rods for angles from 0° or 45° (Refs. 11–15) or relocation[16] of inclusions. These studies showed that modifications in the dispersion

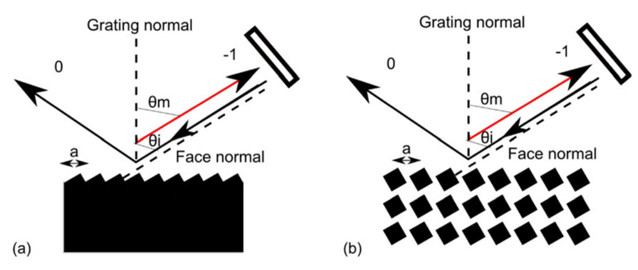

FIG. 1. (a) Littrow condition for a diffraction grating. (b) Setup for blazing measurements. Numbers 0 and -1 correspond to the diffraction orders.

relation for acoustic waves occur, in particular, the formation of new band gaps. As a remark, one should be careful not to make conclusions about band gaps based only on the classical Γ–X–M–Γ path in the first Brillouin zone (BZ), because the symmetry of the unit-cell is broken by the rotation of the inclusion.[17] Band structures should be computed into the first BZ taking into account only of the $\pi/2$-rotation invariance.

The design of the blazed PnC grating combines several rules. First, the blazing angle is selected, which also defines the Littrow incidence angle. We selected steel rods with square cross-section to maximize the facet surface. Each rod is rotated by 30° around the center of the unit-cell. As a result, we have two possible blazing angles, $\theta_{B1} = 60°$ and $\theta_{B2} = -30°$, as depicted in Fig. 2(a). The operating frequency is accordingly defined by Eq. (1) with m = −1. For the blazing angles just defined, operating frequencies are $f_{B1} = 0.329$ MHz and $f_{B2} = 0.569$ MHz. Next, we computed band structures for relevant incidence angles using the finite element method (FEM)[17] for the infinite-size PnC. This step allowed us to locate possible band gaps and to find a configuration for which the blazed frequency lies within a band gap. The band structure in Fig. 2(b) is presented for both the case of normal incidence and for Littrow incidence at −30° and 60°. A complete band gap, valid for all propagation directions, is found to extend from 0.3 MHz to 0.37 MHz, and is depicted as the gray region BG_1. Additionally, directional band gaps appear for normal incidence and are depicted as the light blue regions BG_2 and BG_3. It should be noted that directions −30° and 60° are not symmetry directions of the phononic crystal, in contrast to direction 0°. Finally, in order for blazing frequency f_{B1} to fall within BG_1, period $a = 2.6$ mm is chosen for the demonstration. The square rod width is $r = 1.5$ mm.

FEM simulations of the finite-size PnC immersed in water[10] were then conducted for both one and five layers. The water domain surrounding the PnC is modeled by a finite circular region. A source of pressure waves is modeled by a rectangular solid with three clamped sides and a prescribed normal acceleration on the fourth side. Since a finite PnC and an external source of waves are considered, the finite computational domain is terminated with radiation boundary conditions to avoid unwanted reflections. Fig. 3 shows normalized intensity distributions of diffracted pressure fields for both considered blazing conditions. Acoustic energy is distributed among orders of diffraction 0 and −1 on either sides of the PnC. On the incident side, diffraction to the −1st order is much more efficient than specular reflection to the 0-th order, in accordance with the blazing principle. A comparative analysis of the figures on the transmission side shows that transmission to the 0-th and the −1-st diffraction orders is efficiently cancelled by the 5-layer PnC as compared to the case of the 1-layer grating. It was also verified numerically that the blazing condition is resilient to slight changes in the pitch of the PnC or in the angle of incidence. For instance, a 5% change in the pitch, which is well beyond experimental uncertainty, does not result in any appreciable change in the diffraction efficiency. This numerical observation indicates that experiments are possible even with slightly imperfect structures. It is also clear from Fig. 3 that the 1-layer blazed grating is already quite efficient in theory.

The PnC samples shown in Fig. 4 were then fabricated and diffraction measurements were performed. Metallic plates were perforated to define an array of rotated square holes by laser machining. 100 mm-long square cross-section steel rods were inserted in the holes. Precise periodic alignment of the rods is thus provided by two parallel plates held

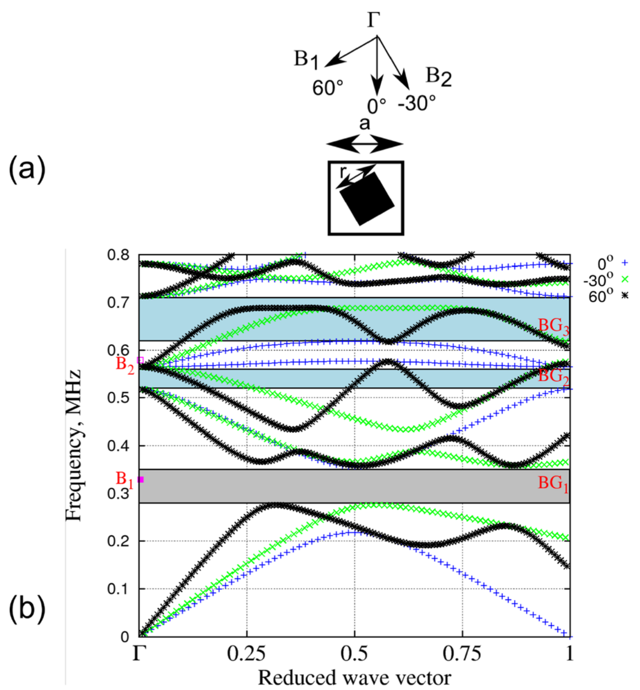

FIG. 2. (a) Definition of the unit-cell of the square-lattice PnC of rotated square steel rods with square cross-section immersed in water. The period is $a = 2.6$ mm and the rod width is $r = 1.5$ mm. Rods are 30°-rotated, so that two blazing angles exist, $\theta_{B1} = +60°$ and $\theta_{B2} = -30°$. (b) Band structure of the PnC for angle of incidence 0°, +60°, and −30°. The blazing frequency for the −1-order of diffraction are $f_{B1} = 0.329$ MHz and $f_{B2} = 0.569$ MHz. BG_1 denotes the complete band gap, while BG_2 and BG_3 are directional band gaps valid only for normal incidence.

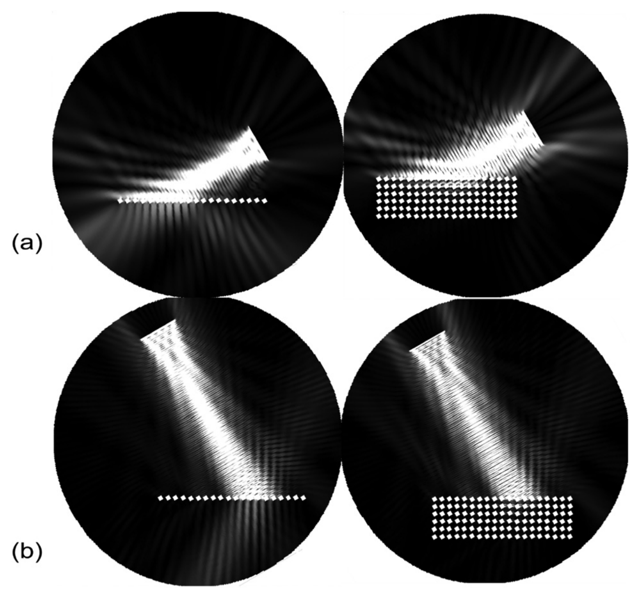

FIG. 3. Finite element simulation of intensity distributions for a (left) 1-layer and (right) a 5-layer PnC. (a) Blazing frequency $f_{B1} = 0.329$ MHz, blazing angle $\theta_{B1} = 60°$. (b) Blazing frequency $f_{B2} = 0.569$ MHz, blazing angle $\theta_{B2} = -30°$.

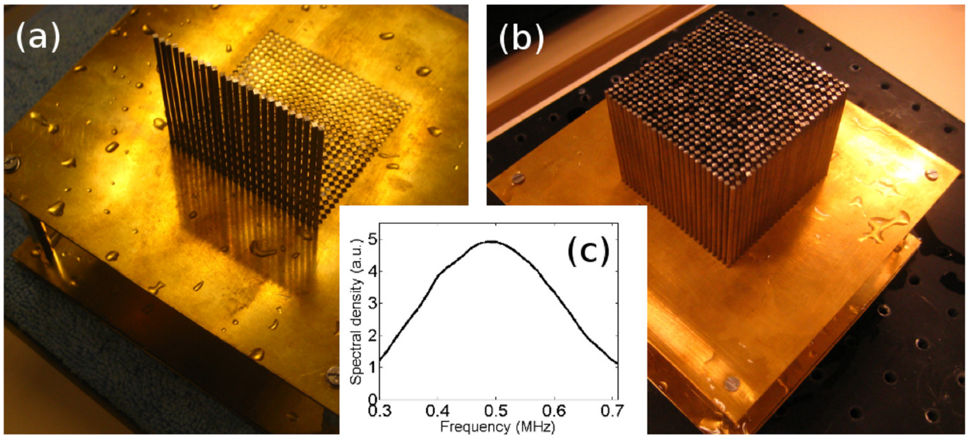

FIG. 4. Photographs of the (a) 1×25 and (b) 25×25 PnC samples used in experiments. (c) Experimentally measured reference spectrum used to normalize transmissions.

at a distance away by post spacers. Experiments were performed using Polar/C-scan system and a short pulse transducer with emission centered at 0.5 MHz.

The reference spectrum, as obtained from a Fourier transform of the short pulse, is shown in Fig. 4(c). The transducer was used as both an emitter and a receiver, by switching the mode of operation after the emission of the pulse. The angle of incidence of the sound beam was varied from $-70°$ to $70°$, with an angular resolution of $0.1°$. Two different PnC samples were considered during the experiments, either a 25×25 array of rods or a 1×25 array. Care was taken that incidence conditions are close to those of plane-wave diffraction. The beam width of the transducer is 24 mm, significantly smaller than the crystal width. Thus, diffraction of the incident beam from the metallic bottom plate can be safely neglected. Pressure waveforms are collected for all angles of incidence for a duration of 400 μs every 20 ns. Frequency analysis is then performed by computing Fourier transform of waveforms in order to determine frequency as a function of angle. Measurements are thus presented in the form of angular spectrograms in Fig. 5, with normalization to the transducer spectrum. The usable frequency range extends from 0.3 to 0.7 MHz.

It is clear that the diffraction efficiency in Fig. 5 is maximal when the Littrow condition is achieved, i.e., it concentrates along curves in the (θ, f) plane that follow Eq. (1). This is a clear evidence that the grating law applies to determine the direction of waves diffracted in water, irrespective of the internal details of the PnC. The normal incidence case $(\theta = 0)$ shows that enhancement of the redirected energy is obtained within band gaps BG_1 and BG_2, though not within BG_3. Experimental data also confirm that the one layer grating is already quite efficient, as the numerical simulations of Fig. 3 suggested. Surprisingly, the diffraction efficiency for $m = -1$ approaches 100% for angles of incidence between $-40°$ and $-70°$, and also around $50°$, even though blazing was not especially expected to operate under such conditions. The diffraction efficiency at blazing points B_1 and B_2 is around 60%, as shown in Table I. It is definitely enhanced by using more layers in the PnC. In the absence of a phononic band gap, at point B_2, the diffraction efficiency is increased to 77% experimentally. Within the complete phononic band gap BG_1, at point B_1, the diffraction efficiency is increased up to 98%. In this case, the 25-layer blazed PnC grating achieves an enhancement factor 1.59 at B_1.

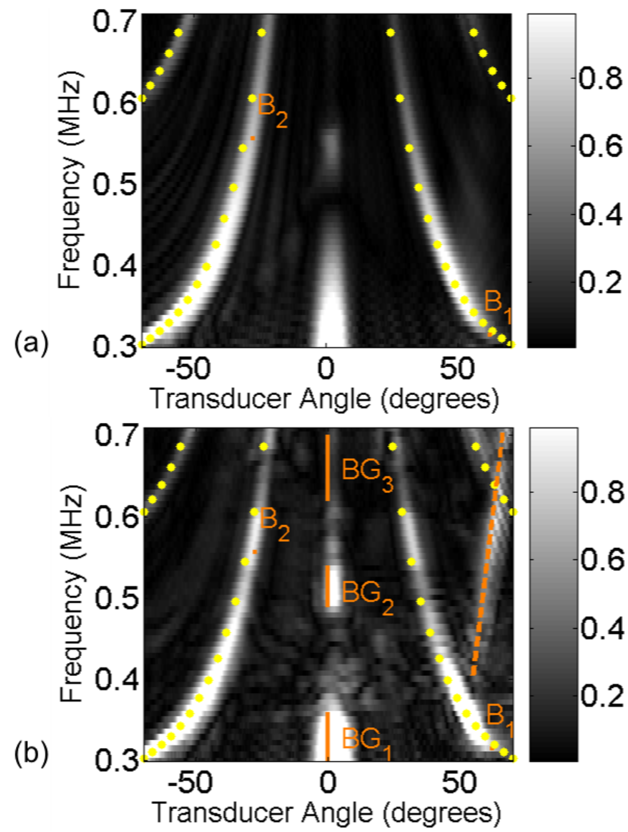

FIG. 5. Normalized angular spectrograms measured when the incidence angle is varied from $-70°$ to $70°$ for the (a) 1×25 and the (b) 25×25 PnC sample. The blazing point B_1 and B_2 are shown for reference. The complete band gap BG_1 and the normal incidence band gaps BG_2 and BG_3 are also depicted. The yellow dotted lines indicate the Littrow condition.

An unexpected linear branch was also observed in the experimental angular spectrograms reported in Fig. 5, extending from $50°$ to $70°$ in the angular range and from 0.45 MHz to 0.7 MHz in the frequency range. We show for completeness in Fig. 6 the temporal measurements for the

TABLE I. Diffraction efficiency of the blazed PnC grating with one or 25 layers, and improvement brought by the PnC thickness.

	Blazing frequency (MHz)	η_1	η_{25}	η_{25}/η_1
B_1	0.329	0.613	0.98	1.59
B_2	0.569	0.6	0.77	1.27

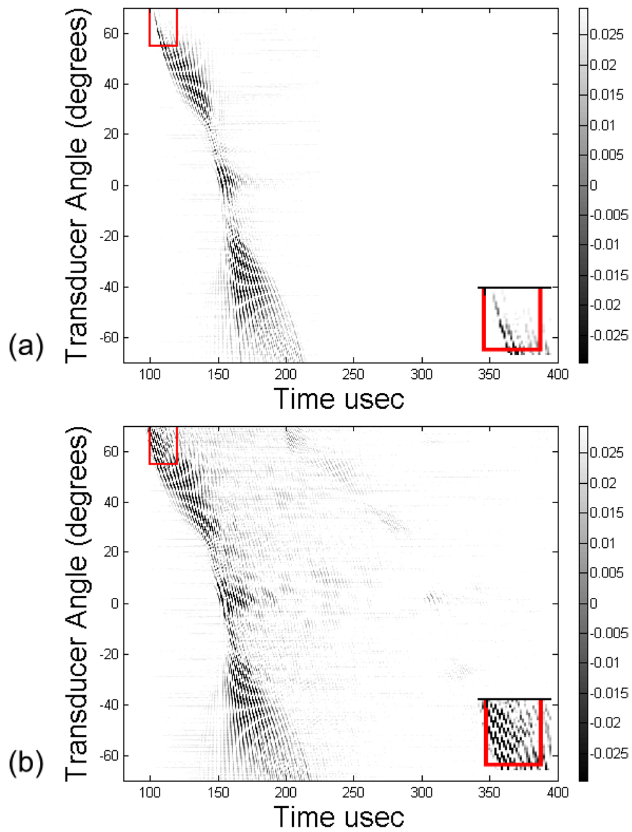

FIG. 6. Time diagrams when incidence angle is varied from −70° to 70° for the (a) 1 × 25 and the (b) 25 × 25 PnC sample. The red rectangles outline the location of the signal which is localized on the dashed orange line of the angular spectrogram in Fig. 5(b).

1-layer and the 25-layer PnC samples. They clearly show the dispersion (mostly delays) that results from coupling to the Bloch waves of the PnC. Their further analysis shows that the linear branch of the 25-layer PnC is given by the signal shown inside a red rectangle in Fig. 5(b), which diffracts with almost no delay as compared to the 1-layer PnC grating case. We were, however, not able to fully understand the origin of this signal and leave its physical explanation to further analysis.

We have presented in this letter the first experimental study of a PnC with rotated rods of square cross-section providing a blazing effect when used as a diffraction grating. We have obtained an enhanced diffraction efficiency in the Littrow configuration when blazing is combined with a phononic band gap, with a maximal experimental efficiency of 98%. We also observed experimentally that a single layer PnC could yield diffraction efficiencies approaching 100% even away from the strict blazing angle of incidence. The condition to have the blazing effect together with a phononic band gap at a given frequency and angle can be rather constraining. Since diffraction on the PnC is mainly governed by the first layer,[10] an alternative configuration could be to combine a PnC with known band gap and a periodic first layer chosen so as to create blazing. This investigation opens the way toward the design of efficient phononic crystal gratings and acoustic wave convertors.

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