Superluminal asymptotic tunneling times through one-dimensional photonic bandgaps in quarter-wave-stack dielectric mirrors

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It is shown that the time needed for light to pass through the optical barrier associated with an antiresonant quarter-wave-stack dielectric mirror, as measured by the group-delay, or phase time, asymptotically reaches a limit that is independent of the barrier thickness and hence of the number of layers. This limit, which scales as the inverse of the refractive-index difference between successive layers, is equal to the mean value of the asymptotic group delays needed for light to reflect off each side of the barrier. This superluminal transmission does not violate causality, as the transmitted intensity is always lower than the intensity that would have been transmitted in vacuum in the absence of the barrier.
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1. INTRODUCTION

It has been shown first theoretically[1,2] and experimentally[3,4] that the time needed for a particle to pass through an opaque barrier by the tunnel effect asymptotically reaches a limit that is independent of the barrier thickness. It has also been shown experimentally[3,4] that the time measure that best describes the arrival time of a wave packet peak is the phase time, or group delay, rather than the semiclassical time of Büttiker and Landauer[5] or the Larmor time.[6] Furthermore, the quantum approach that involves the time-independent Schrödinger equation and the wave approach that involves the Helmholtz monochromatic wave equation are formally equivalent.[7] Based on this equivalence, the electronic tunnel effect and the frustrated optical transmission phenomena were shown to be related.[7,8] Indeed, two kind of barrier must be distinguished: barriers that involve frustration phenomena and result in evanescent waves and barriers that involve anti-resonant phenomena and result in periodic sinusoidal waves. For the first kind of barrier the superluminal group delay is dispersive, usually weakly positive, and remains constant for thick barriers[7–9]; however, weakly negative group delays have been shown to occur in certain instances.[10] For the second kind the group delay is stationary and hence nondispersive, and it reaches a finite asymptotic limit for thick barriers.[11]

In this paper we consider the tunneling of optical pulses through the photonic bandgaps associated with quarter-wave-stack dielectric mirrors as studied experimentally by Steinberg *et al.*[3] and then by Spielmann *et al.*[4] We obtain simple analytical expressions for the asymptotic group delay on transmission and reflection. The theoretical predictions are shown to be supported by the experimental results of Spielmann *et al.*[4]

2. THEORY

We consider a stack of alternate layers with refractive indices n_1 and n_2, with respective thicknesses d_1 and d_2, deposited upon a substrate with refractive index n_s (Fig. 1). The incident medium has refractive index n_0. According to the matrix formulation of Abelès,[12,13] the reflection and transmission of plane waves by the stack are characterized by a 2×2 matrix M such that

$$\begin{bmatrix} E_0{}^+ + E_0{}^- \\ \eta_0(E_0{}^+ - E_0{}^-) \end{bmatrix} = M \begin{bmatrix} E_s \\ \eta_s E_s \end{bmatrix}, \quad (1)$$

where $E_0{}^+$ and $E_0{}^-$ are, respectively, the incident and the reflected electric fields in the incident medium and E_s is the transmitted electric field in the substrate. The following notation has been used following Abelès: η denotes the effective index of refraction for a plane wave propagating with an angle θ in a medium with refractive index n and is given by

$$\eta = \begin{cases} n \cos \theta, & s \text{ polarization} \\ n/\cos \theta, & p \text{ polarization} \end{cases}. \quad (2)$$

Whatever the stack, matrix M assumes the form

$$M = \begin{bmatrix} A & iB \\ iC & D \end{bmatrix}, \quad (3)$$

and the amplitude transmission and reflection coefficients are given, respectively, by

$$t = \frac{E_s}{E_0{}^+} = \frac{2\eta_0}{(\eta_0 A + \eta_s D) + i(\eta_0 \eta_s B + C)}, \quad (4)$$

$$r = \frac{E_0{}^-}{E_0{}^+} = \frac{(\eta_0 A - \eta_s D) + i(\eta_0 \eta_s B - C)}{(\eta_0 A + \eta_s D) + i(\eta_0 \eta_s B + C)}. \quad (5)$$

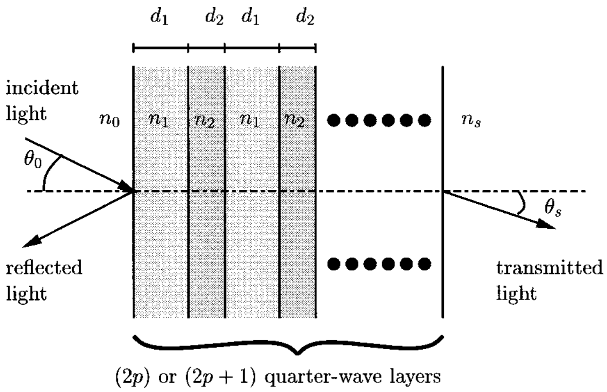

Fig. 1. Stack of alternate layers with refractive indices n_1 and n_2 and with respective thicknesses d_1 and d_2.

From these amplitude coefficients the group delay on reflection and on transmission can be obtained directly once the matrix M is known. The influence of a single layer in the stack, with refractive index n and thickness d, is characterized by the matrix

$$M = \begin{bmatrix} \cos\beta & \frac{i}{\eta}\sin\beta \\ i\eta\sin\beta & \cos\beta \end{bmatrix}, \tag{6}$$

with

$$\beta = \omega(d/c)n\cos\theta = \omega\tau, \tag{7}$$

where ω is the optical frequency and θ is the refraction angle inside the layer. When the optical thicknesses of the layers in the stack are close to quarter-waves, i.e., $nd\cos\theta \approx (2q+1)\lambda_0/4$, where λ_0 is the wavelength in vacuum at optical frequency ω_0 and q is the order of the quarter-wave stack, the matrix M for each single layer expanded to first order with respect to $\Delta\omega = \omega - \omega_0$ becomes

$$M = (-1)^q \begin{bmatrix} -\Delta\omega\tau & \frac{i}{\eta} \\ i\eta- & \Delta\omega\tau \end{bmatrix}. \tag{8}$$

In Eq. (8), τ is independent of frequency and is the traversal time for a single quarter-wave layer at frequency ω_0. Furthermore, the order of the quarter-wave-stack, q, is assumed to be the same for all layers. The influence of two successive layers with respective indices of refraction n_1 and n_2 is given by the product of their matrices:

$$M = M_1M_2 = -\begin{bmatrix} \eta_2/\eta_1 & i\left(\frac{1}{\eta_1} + \frac{1}{\eta_2}\right)\Delta\omega\tau \\ i(\eta_1 + \eta_2)\Delta\omega\tau & \eta_1/\eta_2 \end{bmatrix}. \tag{9}$$

This calculation can be repeated easily for the subsequent layers, and the result is given in Appendix A. We are interested in the asymptotic values of the group delay as the number of layers becomes large. As is detailed in Appendix A, two different cases must be considered: $\eta_1 > \eta_2$ and $\eta_2 > \eta_1$. When $\eta_1 > \eta_2$, the following asymptotic group delays are obtained:

$$(t_D{}^T)_{\text{even}} = \frac{\eta_0\eta_s + \eta_1\eta_2}{\eta_s(\eta_1 - \eta_2)}\tau, \tag{10}$$

$$(t_D{}^T)_{\text{odd}} = \frac{\eta_0 + \eta_s}{\eta_1 - \eta_2}\tau, \tag{11}$$

$$(t_D{}^R)_{\text{even}} = (t_D{}^R)_{\text{odd}} = \frac{2\eta_0}{\eta_1 - \eta_2}\tau. \tag{12}$$

When $\eta_2 > \eta_1$, the same asymptotic group delays now become

$$(t_D{}^T)_{\text{even}} = \frac{\eta_0\eta_s + \eta_1\eta_2}{\eta_0(\eta_2 - \eta_1)}\tau, \tag{13}$$

$$(t_D{}^T)_{\text{odd}} = \frac{(\eta_0 + \eta_s)\eta_1\eta_2}{\eta_0\eta_s(\eta_2 - \eta_1)}\tau, \tag{14}$$

$$(t_D{}^R)_{\text{even}} = (t_D{}^R)_{\text{odd}} = \frac{2\eta_1\eta_2}{\eta_0(\eta_2 - \eta_1)}\tau. \tag{15}$$

Here the subscripts even and odd refer to $(2p)$ and $(2p + 1)$ alternated quarter-wave layers, respectively.

3. DISCUSSION

On examination of the relations obtained in Section 2, we can make the following remarks: First, the asymptotic group delays on reflection are identical for even and odd numbers of layers and do not depend on the refractive index of the substrate. This result is logical because the reflected photons can never see the rear part of the stack. Second, in the general case, the group delay on transmission is equal to the mean value of the group delay on reflection for light coming from the left side of the barrier in a material of index n_0 and of the group delay on reflection for light coming from the right side of the barrier in a material of index n_s. The stack can then be replaced schematically by two reflecting planes that are distances $\delta_0 = (ct_D{}^R)_0/(2n_0\cos\theta_0)$ and $\delta_s = (ct_D{}^R)_s/(2n_s\cos\theta_s)$, respectively, from each side of the stack; reflection then occurs on each of these reflecting planes, and transmission is instantaneous between the two reflecting planes (Fig.

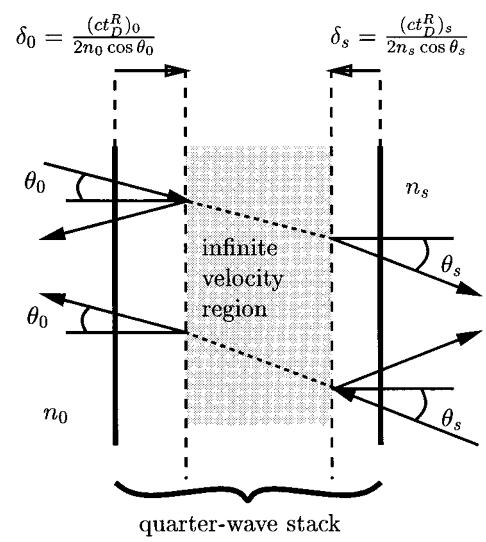

Fig. 2. Equivalent reflection planes describing the reflection from and transmission through the quarter-wave-stack.

2). The relation between reflection and transmission times was derived for an arbitrary stack by Steinberg *et al.* in Ref. 14, where the case of absorption was also considered.

For example, for a central wavelength $\lambda_0 = 800$ nm and $q = 0$, for normal incidence and when the stack consists of an odd number of alternating layers of fused silica (SiO_2; $n = 1.45$) and titanium dioxide (TiO_2; $n = 2.315$) surrounded by air ($n_0 = n_s = 1$), the asymptotic tunneling time ($t_D{}^T = t_D{}^R$) is 1.54 fs if the first layer is TiO_2 and 5.18 fs if the first layer is SiO_2. In the same conditions, except with an even number of layers, the asymptotic tunnel time $t_D{}^T$ is 3.36 fs, i.e., the mean value of the two previous group delays.

It is useful to compare the group delay of the tunneling optical pulse $t_D{}^T$ with that of the incident pulse that would have propagated in the incident medium for a distance given by the physical thickness of the barrier δ, i.e., in the absence of the barrier. This group delay, $t_0{}^T$, which is simply given by $n_0 \cos \theta_0 \delta / c$, is the vacuum time when $n_0 = 1$. Note that $t_0{}^T$ is proportional to the barrier thickness and hence becomes infinite as this thickness increases.

Figure 3 shows the evolution of the tunneling times $t_D{}^T$ and $t_D{}^R$, obtained by numerical simulation, as a function of the number of layers for even and odd numbers of alternating layers of SiO_2 and TiO_2 deposited upon a fused-silica substrate ($n_s = 1.45$) for normal incidence and $q = 0$. The asymptotic values of these group delays as given by relations (10)–(15) are also shown. It can be seen that the asymptotic limits are almost reached after only 10 quarter-wave layers. Figure 4 specifies the evolution of the asymptotic tunneling time $t_D{}^T$ as a function of the angle of incidence θ in air for s and p polarizations and for the quarter-wave-stack of Fig. 3(a). The asymptotic tunneling time depends only slightly on the angle of incidence below 40°, but the variation becomes dramatic for s polarization above that value. Note that for this computation the optical thicknesses of the layers are kept equal to a quarter-wave for all angles of incidence. This critical dependence of the tunneling time on polarization and angle of incidence was studied experimentally by Steinberg *et al.*[15] Their results cannot, however, be compared directly with our Fig. 4 because obviously the thicknesses of the layers were constant in their experiments, whereas our asymptotic expressions require that the layers remain quarter-waves and hence their thicknesses depend explicitly on the angle of incidence.

The experimental data obtained by Spielmann *et al.*[4] offer a direct test of the theoretical results presented above. Spielmann *et al.* used quarter-wave-stack dielectric mirrors of increasing thickness with a structure of (substrate) $(HL)^p$(air), where H and L represent, respectively, a TiO_2 and a SiO_2 quarter-wave ($\lambda_0 = 800$ nm and $q = 0$), and with $p = 3, 5, 7, 9, 11$. These five different samples were illuminated with p-polarized 28-THz-bandwidth femtosecond pulses centered on $\lambda_0 = 800$ nm with an angle of incidence of 20° from air. Using a time-of-flight measurement apparatus, Spielmann *et al.* were able to measure directly the delay difference $\Delta\tau = t_D{}^T - t_0{}^T$ between the tunneling pulse and the pulse transmitted in the absence of the barrier with a temporal resolution better than 0.3 fs. Figure 5 reproduces these experimental results (Fig. 3 of Ref. 4). From these results it is apparent that the tunneling becomes superluminal starting from sample $(HL)^7$ and that the predicted delay differences were globally overestimated. Our predicted

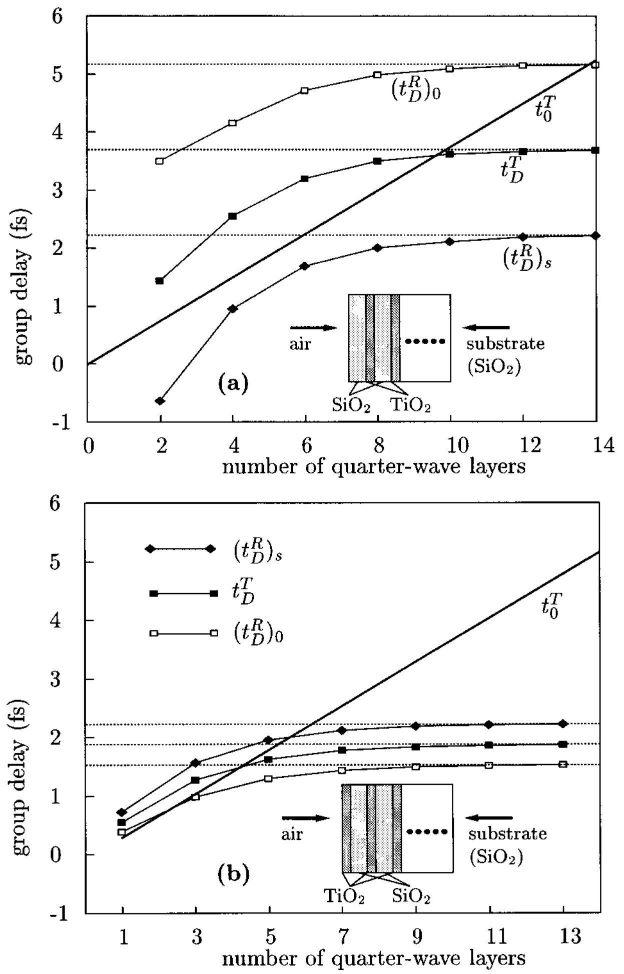

Fig. 3. Group delays as a function of the number of layers for (a) ($2p$) and (b) ($2p + 1$) quarter-wave layers, assuming normal incidence, $\lambda_0 = 800$ nm, and $q = 0$. Horizontal lines, the asymptotic limits of Eqs. (10)–(15). Transmission is superluminal as soon as $t_D{}^T$ is smaller than $t_0{}^T$.

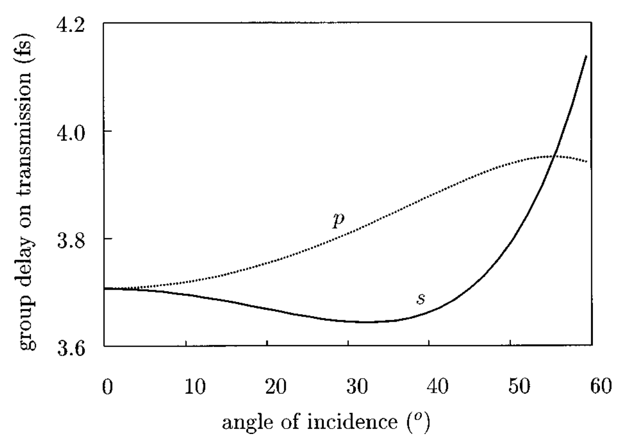

Fig. 4. Dependence of the asymptotic tunneling time on the angle of incidence for the quarter-wave stack of Fig. 3(a), $\lambda_0 = 800$ nm, and $q = 0$.

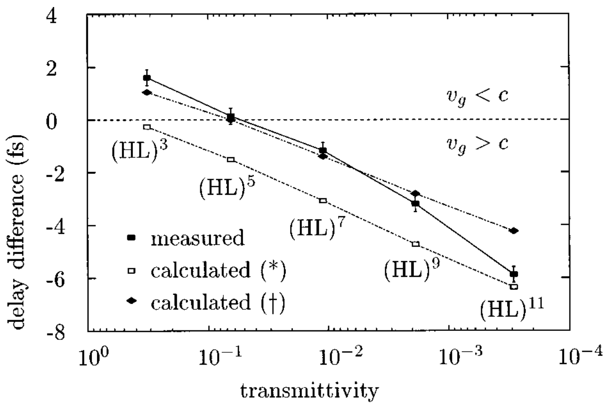

Fig. 5. Measured and calculated (*) delay difference $\Delta\tau$ (20° incidence, p polarization, $\lambda_0 = 800$ nm and $q = 0$) in the experiment of Spielmann *et al.*[4] Calculated (†) $\Delta\tau$ with the present theory.

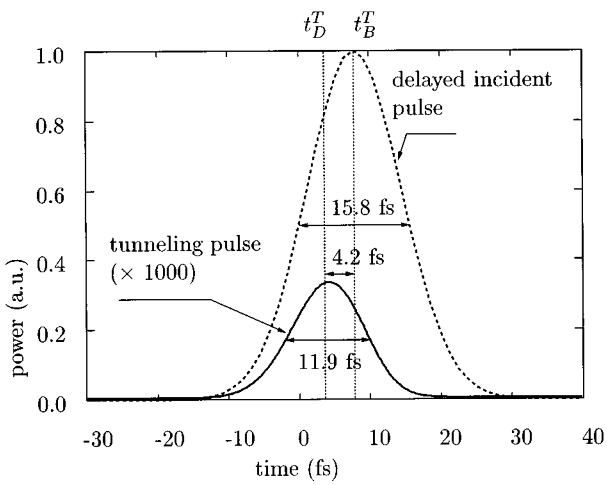

Fig. 6. Numerical simulation of the transmission of a 28-THz-bandwidth Gaussian pulse through barrier (HL)^{11} of Fig. 5 (20° incidence, p polarization, $\lambda_0 = 800$ nm, and $q = 0$).

delay differences, also shown in Fig. 5, are consistent with the first four measurements. As discussed qualitatively by Spielmann *et al.*,[4] the fact that the group velocity is superluminal does not imply that causality is violated. This point is illustrated by the numerical simulation of Fig. 6, which shows that the instantaneous power of a Gaussian pulse transmitted through sample $(HL)^{11}$ never precedes that of the pulse that would have been transmitted in the absence of the barrier (note that a magnification factor of 1000 was applied to the tunneling pulse). Such an argument can obviously be valid only with classic wave packets. From a quantum point of view, there remains a non-nil probability that a single photon would have crossed the point of the barrier faster than it would have in the absence of the barrier although the use of the phase time has been questioned for such a tunneling process (see, e.g., Refs. 16–19 for a discussion of this topic and more references). It can also be observed that the tunneling pulse is shorter than the incident pulse; here an incident 15.8-fs FWHM pulse gives rise to a 11.9-fs FWHM pulse. This effect was observed experimentally by Spielmann *et al.*[4] and was explained simply by the spectral reshaping of the pulse by the spectral transmission of the barrier.

4. CONCLUSION

We have obtained simple explicit relations that give the asymptotic group delays on reflection and transmission introduced by an optical barrier consisting of a stack of alternating quarter-wave layers, which constitutes a one-dimensional photonic bandgap. These expressions are consistent with tunneling times on transmission previously measured experimentally. We have also observed that the traversal time of such a barrier is the mean group delay needed for light to reflect from the left and right sides of the barrier. This observation should make the measurement of tunneling times much easier because the reflection factors are always close to unity, whereas transmission drops dramatically with barrier thickness.

APPENDIX A

From the reflection and transmission amplitude coefficients of Eqs. (4) and (5) it is easy to obtain that the phase retardation on transmission is given by

$$\tan \Phi_T = \frac{\eta_0 \eta_s B + C}{\eta_0 A + \eta_s D}, \tag{A1}$$

whereas the phase retardation on reflection is given by

$$\tan(\Phi_R - \Phi_T) = -\frac{\eta_0 \eta_s B - C}{\eta_0 A - \eta_s D}. \tag{A2}$$

Furthermore, it can be shown that the influence of $(2p)$ alternating quarter-wave layers with respective indices of refraction n_1 and n_2 is given by

$$M = (M_1 M_2)^p = (-1)^p \begin{bmatrix} (\eta_2/\eta_1)^p & i\,\dfrac{a_p}{\eta_1 - \eta_2}\,\Delta\omega\tau \\ i\,\dfrac{\eta_1\eta_2 a_p}{\eta_1 - \eta_2}\,\Delta\omega\tau & (\eta_1/\eta_2)^p \end{bmatrix}, \tag{A3}$$

with

$$a_p = (\eta_1/\eta_2)^p - (\eta_2/\eta_1)^p, \tag{A4}$$

whereas the influence of $(2p + 1)$ alternating quarter-wave layers is given by

$$M = (M_1 M_2)^p M_1 = (-1)^{p+q+1} \begin{bmatrix} b_p \Delta\omega\tau & -\dfrac{i}{\eta_1}(\eta_2/\eta_1)^p \\ -i\eta_1(\eta_1/\eta_2)^p & b_p \Delta\omega\tau \end{bmatrix}, \tag{A5}$$

with

$$b_p = \frac{(\eta_1/\eta_2)^{p+1/2} - (\eta_2/\eta_1)^{p-1/2}}{(\eta_1/\eta_2)^{1/2} - (\eta_2/\eta_1)^{-1/2}}. \tag{A6}$$

Using matrices (A3) and (A5) and expressions (A1) and (A2), we can compute the phases Φ_T and Φ_R as a function of the frequency difference $\Delta\omega = \omega - \omega_0$, and then obtain and the group delays on transmission $[t_D{}^T$

$= (\partial\Phi_T)/\partial\omega]$ and reflection $[t_D{}^R = (\partial \Phi_R)/\partial\omega]$. We are interested in the asymptotic values as the number of layers becomes large, i.e., as p tends to infinity. Two different cases must be considered: $\eta_1 > \eta_2$ and $\eta_2 > \eta_1$. When $\eta_1 > \eta_2$ and p is large, $(\eta_2/\eta_1)^p$ can be neglected against $(\eta_1/\eta_2)^p$, and the different group delays of Eqs. (10)–(12) follow. When $\eta_2 > \eta_1$ and p is large, $(\eta_1/\eta_2)^p$ can be neglected against $(\eta_2/\eta_1)^p$, and the group delays of Eqs. (13)–(15) follow.

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