Guided elastic waves along a rod defect of a two-dimensional phononic crystal

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(Received 19 November 2003; published 11 June 2004)

The propagation of classical waves in periodic structures is receiving increasing attention as a generic procedure for obtaining innovative or enhanced functions in passive components dedicated to signal processing. Most studies have been focused on photonic crystals, for which the appearance of frequency gaps for the propagation of electromagnetic waves has been demonstrated both theoretically and experimentally, and which are used for the achievement of new optical devices. In parallel, the propagation of elastic and acoustic waves in periodic structures made of materials with different elastic properties, also called phononic crystals, is receiving a growing interest. For instance, in the case of elastic waveguides, bulk localized states have been predicted [1,2] while surface states as well as localization phenomena have been calculated and observed in linear and point defects [3]. Localized states have also been investigated in relation to planar defects [4].

In a recent work [5], we investigated the out-of-plane propagation of elastic waves in a two-dimensional phononic band-gap material composed of quartz rods embedded in an epoxy matrix. Band-gaps for nonzero values of the wave-vector component parallel to the rods were shown to exist. It is essential for applications of a such a two-dimensional periodic structure, that displays an absolute band-gap in its phononic band structure for in-plane propagation, to know the extent to which acoustic waves can propagate out of plane while an in-plane absolute band-gap can still be seen in the corresponding band structure. Also, the possibility of guiding waves propagating perpendicularly to the plane of the structure is revealed by such an analysis. The presence of a piezoelectric material offers the possibility of exciting and detecting waves electrically in the structure using the piezoelectric effect. Interestingly, the frequencies of elastic modes are in general only slightly affected by piezoelectric coupling, that is less than 1% even for rather strong piezoelectrics. The occurrence of frequency gaps and localized modes is then very slightly modified by piezoelectricity. In this paper, we investigate theoretically the propagation of guided elastic waves in a two-dimensional periodic structure, that consists of an array of infinitely long parallel square-section rods of tungsten embedded in an epoxy matrix. The advantage of a metal such as tungsten with respect to a material such as quartz lies in the much higher impedance ratio with epoxy, which results in the formation of full-band-gaps with lower filling fractions, or of larger band-gaps for the same filling fraction. The intersections of the rods’ axes with the perpendicular plane form a two-dimensional Bravais lattice. To achieve out-of-plane propagation with a confinement in the (x,y) plane, we add a piezoelectric rod defect, in this case aluminum nitride (AlN), with the intent of breaking the periodicity and inducing localized modes in the band-gaps. The AlN C-axis is directed along the z-axis since this configuration is the most favorable for the excitation of compressional waves. Numerical calculations are performed using a plane-wave-expansion (PWE) method, which was originally developed for 1-3 connectivity piezoelectric composites [6] and further adapted to anisotropic solid-solid phononic band-gap materials [5].

Figure 1(a) displays the cross-section of the phononic crystal considered in this work. This phononic crystal is composed of tungsten rods in an epoxy matrix. The tungsten inclusions are arranged periodically on a square lattice and are assumed to have a square cross-section so that the filling

![FIG. 1. (a) Cross-section of a bi-periodic solid-solid phononic band-gap material, consisting of tungsten rods in epoxy. (b) Cross-section of a super-cell solid-solid phononic band-gap material, consisting of AlN rod defect in tungsten/epoxy structure.](image-url)
fraction \((d/a)^2\) is 0.20. For instance, the width of the inclu-
sions \(d=45 \mu m\) for a lattice parameter \(a=100 \mu m\). This
choice of materials provides a very high scattering contrast
since the ratio between the longitudinal acoustic impedances
of tungsten and epoxy is around 35. Table I displays the
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the band structures within the limitation related to computa-
tional requirements. Our aim is to obtain a good physical description of
phononic crystal will considerably increase. Therefore, in
order to obtain the same convergence properties as for the perfect
phononic crystal, in order to retain the same convergence properties.
In order to achieve waveguiding along the rods, we break
the periodicity of the structure by introducing a defect. This
is produced by replacing one tungsten rod by one aluminum
nitride (AlN) rod, as depicted in Fig. 1(b). The width of the
square section of the AlN rod is the same as for tungsten
rods, i.e., 45 \(\mu m\) for a period of 100 \(\mu m\). The longitudinal
acoustic impedance of AlN is ten times larger than that of
epoxy, although it is three times less than that of tungsten.
For practical computations, a super-cell is defined so that the
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For practical computations, a super-cell is defined so that the
AlN/tungsten/epoxy structure can still be investigated with
the PWE method. The super-cell is a \(3\times3\) tungsten-rod array
with the central rod replaced by an AlN rod, as depicted in
Fig. 1(b). Since the super-cell is three times larger than the
elementary cell, \(18\times18\) Fourier and Bloch-Floquet harmon-
ics are used for computations with the defective phononic
crystal, in order to retain the same convergence properties.
In order to investigate the changes induced by the pres-
ence of the AlN rod defect, Fig. 3 displays the dispersion
formula for modes that propagate inside the structure with a
propagation constant \(\gamma_c=0.1\). Four band-gaps are apparent.
From low to high frequencies, the first, third and fourth

\[
\gamma_c = k_c a/2\pi = a/\lambda_z
\]

![FIG. 2. Projection of the phononic band structures in the \((k_x,k_z)\)
plane onto the \((k_x,f)\) plane, for the tungsten-epoxy phononic
crystal. Delimited white regions indicate absolute stop-bands in the
\((k_x,k_z)\) plane.](image-url)
band-gaps are labeled (a), (b) and (c), respectively in Fig. 2. When introducing the defect rod, four flat branches — labeled F, C1, T, and C2 — appear in the band-gap regions. Three of them appear in the third band-gap and one in the fourth. Whatever the in-plane wave-vector, there is no dispersion for these defect modes, i.e., they are localized in the $s_x$, $s_y$ plane around the defect region with a transverse group velocity $s_d v/dk_d$ equal to zero. This is a clear indication that guided modes exist for frequencies inside the band-gaps. In order to illustrate the confinement of defect modes, the eigenvectors for all flat branches are plotted in Figs. 4 and 5. Among these maps, the F and T modes depicted in Fig. 4 have mostly an in-plane polarization. Indeed, mode F depicted in Fig. 4(a) is the fundamental flexural mode of the rod defect with only components $u_x$ and $u_y$ in the $s_x$, $s_y$ plane. This mode is in fact degenerate because of the in-plane isotropy of the structure. Similarly, mode T depicted in Fig. 4(b) is a torsional mode with components $u_x$ and $u_y$, that is localized around the rod defect. Modes C1 and C2 are depicted in Fig. 5. These modes are of the longitudinal or compressional type, with mostly a $u_z$ component, and they are localized inside and in the vicinity of the rod defect. Mode C1, depicted in Fig. 5(a), is well confined inside and around the AlN rod. Mode C2, depicted in Fig. 5(b), has a more complex structure as the displacements of the AlN rod and of the epoxy interstice are in opposite phase.

It is remarkable that the modes that have been found are exactly of the same types as those found in classical elastic waveguides [7], i.e., we identified flexural, torsional and compressional modes. In classical elastic waveguides, the boundary conditions are responsible for the apparition of a discrete number of guided modes. In the present case, the phononic crystal surrounding the defect rod plays a similar role for frequency intervals within which an absolute band-gap exists.

The four guided modes described above are spatially localized in the defect region in the $(x,y)$ plane, but they are propagative in the $z$ direction. Figure 6 shows the band structures in the $(k_x,k_y)$ plane projected onto the reduced-frequency normalized-wave-vector plane, for the tungsten/epoxy/AlN structure. The white regions again indicate absolute band-gaps in the $s_x$, $s_y$ plane. It can be noticed that the band-gaps obtained with the supercell are almost identical to those obtained with the elementary cell. This demonstrates that the band-gap properties of the phononic crystal are not significantly perturbed by the rod defect with only components $u_z$ in the $(x,y)$ plane.

FIG. 3. The phononic band structures in the $(k_x,k_y)$ plane along the $M$-$\Gamma$-$X$-$M$ path for $\gamma_z=0.1$, for the tungsten/epoxy/AlN structure. Defect modes appear in two band-gap regions.

FIG. 4. Relative magnitude of in-plane displacements for (a) the flexural mode, F, with $fa=751.9$ m/s and (b) the torsional mode, T, with $fa=929.0$ m/s for the tungsten/epoxy/AlN structure. $\gamma_z=0.1$ for these computations.

FIG. 5. Relative magnitude of out-plane displacements of (a) the compression mode, C1, with $fa=848.0$ m/s and (b) the compression mode, C2, with $fa=1198.4$ m/s for the tungsten/epoxy/AlN structure. $\gamma_z=0.1$ for these computations.
inclusion of the periodic defect. The branches that now appear inside the band-gaps are those of defect modes and are representative of their dispersion as a function of the $z$ component of the wave-vector. It can be seen that the phononic crystal with a rod defect possesses several guided modes that can exist within the different band-gaps. If $\gamma_z$ is held constant, then these modes give rise to flat branches in the $(k_x, k_y)$ plane as in Fig. 3. Consequently, their group velocities are zero in the transverse plane and they are constrained to propagate along the rod axis. Let us also notice that when $\gamma_z$ decreases from 0.5 to 0, the frequency of the flexural mode in Fig. 3 decreases as well and finally the corresponding dispersion curve merges with a bulk band. At the same time, the other modes (T, C1 and C2) depicted in Fig. 3 change only slightly and the corresponding eigenvectors remain almost unchanged with respect to those shown in Figs. 4 and 5. In practice, to excite a guided mode at a given frequency, the piezoelectric defect rod should be submitted to an applied electric field. Since the defect mode is situated in an absolute band-gap for in-plane propagation, the excitation should remain mainly localized in the vicinity of the defect rod. Nevertheless, it should be mentioned that the phononic crystal does not display any omnidirectional gap for propagation in the three-dimensional space, which means that, for a given frequency, bulk modes of the crystal coexist simultaneously with the guided mode.

In summary, we have computed the phononic band structure of an infinite square array of parallel tungsten rods embedded in an epoxy matrix, forming a two-dimensional phononic crystal. For nonzero values of $k_z$, the phononic crystal possesses absolute band-gaps in the plane perpendicular to the rods, i.e., for all polarizations of elastic waves propagating in the plane of the structure. We have established the occurrence of defect modes by replacing one tungsten rod by an aluminum nitride rod in the tungsten/epoxy composite. These modes can be found inside the band-gaps, with their vibration being localized in the vicinity of the defect sites. They are therefore capable of propagating energy along the rod defect. As a consequence, this study predicts the possibility of achieving solid-solid phononic fibers supporting guided elastic waves along their axis.