Influence of nonoverlapping noise on regularized linear filters for pattern recognition

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We analyze the behavior of regularized linear filters designed for overlapping noise in the presence of images distorted by nonoverlapping noise. In particular we show that there necessarily exist non-null values of the target's illumination that result in the failure of regularized linear filters. The characteristics of those values are analyzed and discussed relative to the correlation length of the background noise.

The study of the detection and restoration of a signal distorted by nonadditive noise has recently become a topic of growing interest in the optical signal processing community. Most of the filters that are used have been designed to be optimal for additive noise, but, although white Gaussian additive noise is an acceptable model for noise corrupting radar signals, it is often inaccurate for describing noise in optical images. In particular, in many images the input scene noise may be spatially disjoint from the target that is to be detected: noise is then said to be nonoverlapping.[1] A typical example of such a situation is the observation of an airplane flying above a soil background. Even if the plane itself is not distorted by noise, the correlation with background may produce spurious peaks, and thus the background can be considered noise.

Javidi *et al.* designed a filter that is optimal in the hypothesis-testing sense in the case of Gaussian nonoverlapping background noise[2]; this filter performed better than the matched filter on images distorted by nonoverlapping noise. Another filter based on Wiener theory was designed[3] and yielded good results. A generalization of the matched filter that leads to a linear filter's optimizing a new metric has also been proposed.[4] Both filters require one to know the average of the background noise, which is in general unknown. To illustrate the difference in behavior of linear filters designed for overlapping noise in the presence of additive and nonoverlapping noise, we first consider the two input image examples shown in Fig. 1. In the lower part of Fig. 1 we show the square modulus of the correlation function of these images with a linear optimal trade-off (OT) filter.[5] It is clear that the linear filtering technique is much more robust for additive noise than for nonoverlapping noise.

It is the purpose of this Letter to characterize this behavior. Let us denote the target t_i, the noise b_i, and the impulse response of the considered linear filter h_i. One-dimensional notations are used for more simplicity. In the Fourier domain, most filters can be written as $\hat{h}_v = \hat{t}_v/\hat{B}_v$, where $\hat{h}_v$ ($\hat{t}_v$) is the Fourier

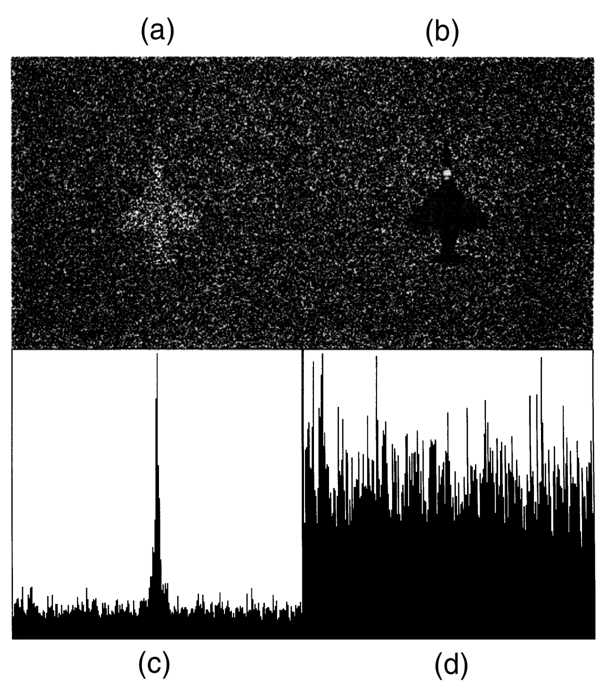

Fig. 1. (a) Scene with the target and additive uniform noise, (b) scene with the target and nonoverlapping uniform noise, (c) correlation of image (a) with an OT filter, (d) correlation of image (b) with the same OT filter. Note that (c) and (d) are plots of the maximum of each line of the modulus square of the correlation plane.

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transform of h_i (t_i) and v is the spatial frequency. $\hat{B}_v$ depends on the linear filter that is considered. For example, $\hat{B}_v = \hat{S}_v$ for the matched filter[6] when the spectral density of the noise model is $\hat{S}_v$. For the phase-only filter,[7] $\hat{B}_v = |\hat{t}_v|$ and $\hat{B}_v = (1 - \mu)\hat{S}_v + \mu|\hat{t}_v|^2$ for the OT filter between noise robustness and peak sharpness. Except for the matched filter in the presence of white noise (which is known to have a very poor discrimination capability), $\hat{B}_v$ generally corresponds to a low-pass filter. Thus $\hat{h}_v$ is the result of the filtering of $\hat{t}_v$ with a high-pass filter. The impulse response of such a filter is shown in Fig. 2. We can see that it is possible to divide this impulse response h_i into two parts, h_i^+ and $-h_i^-$, so that $h_i = w_i h_i^+ - (1 - w_i)h_i^-$, where w_i denotes a unit-magnitude window that defines the support of the target (i.e., $w_i = 1$ within the target and is zero elsewhere). The elements h_i^+ lie in the support of the target and the elements h_i^- outside. Furthermore, the mean value of the filter is generally made equal to zero, which can also be obtained implicitly with OT filters. Although it is not necessary, we will make this assumption in the following since it simplifies the analysis. We thus have $\sum_i h_i = 0$, and then $\sum_i h_i^+ = \sum_i h_i^-$, with $\sum_i h_i^+$ and $\sum_i h_i^-$ both positive. However, we notice that not all elements h_i^+ and h_i^- are positive (see Fig. 2).

The model of an image distorted by nonoverlapping noise can be written as[3]

$$s_i = at_i + b_i(1 - w_i), \tag{1}$$

where s_i is the input image and a is a possible amplitude parameter. For a classical additive noise model, the input image would be $s_i = at_i + b_i$. The central value of the correlation between the input image and the filter can thus be written as $C_{s,h} = \sum_i h_i t_i$. Let us denote a_{non} (a_{add}) the value of a that leads to a zero correlation value in the nonoverlapping (additive) noise case. Thus one has

$$a_{\mathrm{non}} = \frac{\sum_i h_i^- b_i}{\sum_i h_i^+ t_i}, \qquad a_{\mathrm{add}} = -\frac{\sum_i h_i b_i}{\sum_i h_i^+ t_i}. \tag{2}$$

Because b_i and t_i are positive variables, a_{non} is expected to be a positive random variable, whereas a_{add} is a random variable with a zero mean whatever the mean value $\langle b \rangle$ of the additive noise, because $\sum_i h_i = 0$. Thus for nonoverlapping noise there always exists a positive non-null value of the illumination that leads to a null value of the correlation function at the target location. This situation can occur even if the signal-to-noise ratio of the input image {which equals $a^2[\sum_i (t_i)^2]/[\sum_i (b_i)^2]$} is high.

It is important to note that in some cases an approximate localization of the target may be possible even if the value of the correlation function at the target location is zero. This may occur when the filter is not regularized,[8] that is, when its correlation function with the target is very narrow. In this case, for $a = a_{\mathrm{non}}$, the value of the correlation function at the target location is zero by definition, but there may still be a peak just a few pixels away from the exact target location, so that the target is localized approximately. However, such nonregularized filters have been shown to be oversensitive to small variations of scale and orientation of the target.[8] On the contrary, regularized filters are more robust to such variations. Their correlation functions with the target have broader peaks and smoother variations. Thus, in this case, if the value of the correlation function at the target location is zero, there will be no peak near the target location. Because regularized filters are much more useful in practice, we consider only this type of filter in the following.

We now concentrate on the case of nonoverlapping noise. The average of the random variable a_{non} is $\langle a_{\mathrm{non}} \rangle = \langle b \rangle (\sum_i h_i^-)/(\sum_i h_i^+ t_i)$, and its variance is

$$\langle \delta a_{\mathrm{non}}{}^2 \rangle = \frac{\sum_i |\hat{h}_v^-|^2 S_v}{|\sum_i h_i^+ t_i|^2}, \tag{3}$$

where $\hat{h}_v^-$ is the Fourier transform of h_v^- and S_v is the spectral density of the noise, which is supposed to be stationary. If h_v^- is a low-pass filter (which is generally the case), its frequency spectrum is a peak with a maximum for the frequency zero. Thus when the noise becomes correlated, that is, when the S_v function becomes narrower and concentrated on low frequencies, the value of $\langle \delta a_{\mathrm{non}}{}^2 \rangle$ increases. This means that the fluctuations of the value of the illumination leading to a null correlation increase with the correlation length of the noise.

In fact, detection or localization is possible as long as the square modulus of the correlation of the filter with the image at the target location ($|C_{s,h}^0|^2$) is larger than the maximum value of the square modulus of the correlation with the background: $|C_{s,h}^0|^2 > \max_i |C_{b,h}^i|^2$,

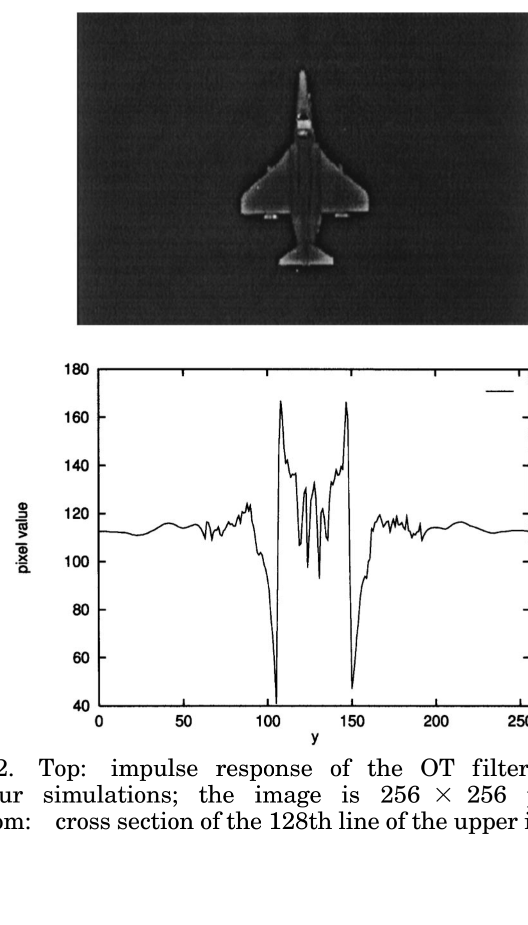

Fig. 2. Top: impulse response of the OT filter used in our simulations; the image is 256 × 256 pixels. Bottom: cross section of the 128th line of the upper image.

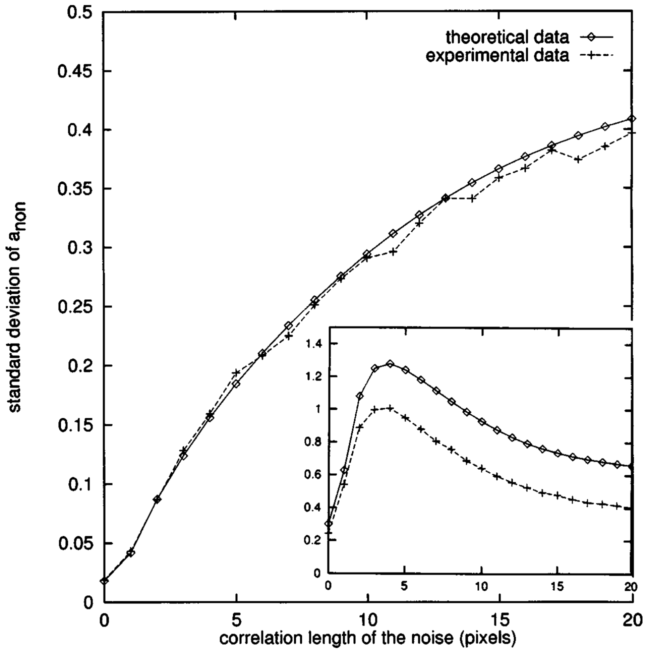

Fig. 3. Square root of $\langle \delta a_{\text{non}}{}^2 \rangle$ computed from Eq. (3) and the standard deviation of a_{non} measured from 500 noise realizations. These two values are plotted versus the correlation length of the noise expressed in pixels. Inset: the upper curve plots $10(\langle |\delta C_{b,h}|^2 \rangle / |\sum_i h_i^+ t_i|^2)^{1/2}$ computed from Eq. (4); the lower curve plots the average length of the forbidden interval measured from 100 noise realizations. These two values are plotted versus the correlation length of the noise expressed in pixels. Note the parallel variations of the curve.

with $C_{b,h}^i = \sum_j h_j b_{j-i}$. The values of a that do not satisfy this inequality will lead to a false or no detection and localization. Let a_m be the minimum of these values and a_M their maximum. We call the interval $[a_m; a_M]$ the forbidden interval. The width of this interval depends on $\max_i |C_{b,h}^i|^2$. One can easily compute the mean $\langle C_{b,h} \rangle = 0$ and the variance $\langle \delta C_{b,h}{}^2 \rangle$ of $C_{b,h}^i$:

$$\langle |\delta C_{b,h}|^2 \rangle = \sum_v |\hat{h}_v|^2 S_v = \sum_v \frac{|\hat{t}_v|^2 S_v}{B_v^2} \cdot \tag{4}$$

The equation shows that the width of the forbidden interval depends on the correlation length if the modulus of the filter is not inpedendent of the spatial frequencies (which would have been the case with the phase-only filter). For example, with bandpass filters such as OT filters, when the noise becomes correlated (i.e., S_v concentrates on low frequencies) the value of $\langle \delta C_{b,h}{}^2 \rangle$ decreases, so that one expects that the length of the forbidden interval will also decrease.

To illustrate these results and conjectures, we now show some numerical simulations of the OT linear filter with input images distorted by nonoverlapping noise. We first consider a 256×256 pixel image of the airplane shown in Fig. 1, whose average brightness is 0.77. We choose as a background noise model a Gaussian noise of 1.0 average and 0.33 standard deviation. The autocorrelation function of the noise is a Gaussian with a width varying from 0 (white noise) to 20 pixels. We first noticed that the mean value of $\langle a_{\text{non}} \rangle$ is nearly constant whatever the correlation length of the background noise and is equal to the theoretical value computed from the first of Eqs. (2). Two curves representing the standard deviation of a_{non} computed from Eq. (3) and experimentally measured on 500 noise realizations for each correlation length are plotted in Fig. 3. One can notice the agreement between the numerical data and theoretical values. The inset of Fig. 3 shows the average width of the forbidden interval measured on 100 noise realizations and $\langle |\delta C_{b,h}|^2 \rangle / |\sum_i h_i^+ t_i|^2$ computed from Eq. (4) plotted versus the correlation length of the noise. As we conjectured above, the two curves have parallel variations and decrease for high correlation lengths.

In conclusion, in the presence of nonoverlapping background noise there is always an interval of target-illumination values for which regularized linear filters designed for overlapping noise fail to detect the target, even if the classic signal-to-noise ratio of the scene is high and the target is not distorted. We have shown how the characteristics of this interval vary with the correlation length of the noise. The purpose of our future research will be to use this information to design detection strategies that are robust to nonoverlapping background noise.

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