Coupling of high-quality-factor optical resonators

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Abstract
We improve theoretically and experimentally the problem of the coupling between a high
Q-factor resonator and its external coupler. We have observed oscillations of ringing induced
by the sweeping of the excitation frequency of an active microsphere. Thanks to this approach,
the quality factor of an optical resonator was measured and we obtained $Q = 5.8 \times 10^8$.

(Some figures may appear in colour only in the online journal)

1. Introduction

Optical resonators with high quality factor (Q-factor) present
great interest for several applications in optoelectronic [1],
metrology or fundamental physics [2, 3]. For instance,
they can be used in optical filtering, all optical switching,
low-threshold nonlinear optics, etc. Main results have been
obtained since 1996 [4]. Recently, results have been obtained
with mono-crystalline resonators such as MgF_2, quartz or
CaF_2 whispering gallery mode (WGM) resonators [5–9].
Rare-earth element-doped microspheres are tiny lasers that
intrinsically offer narrow linewidth emission [10]. A key
feature of the characterization of the resonators is to perform
good coupling. Performing this characterization is quite
challenging because it is a necessary but difficult technique.
That is why we improve theoretically and experimentally the
problem of the coupling between a high-Q-factor resonator
and its external coupler. They are characterized by different
methods such as slow and fast frequency sweeping [11].
Resonators can be coupled to tapered fibers or prisms. We
focus here on characterization techniques of passive optical
resonators or active (erbium-doped ZBLALiP [12, 13]) WGM
resonators in terms of Q-factor determination.

2. Properties of a doped microresonator

Figure 1 shows schematically the WGM microresonator and
its access line. It is possible to isolate each resonance,
although the resonator is inherently multimode. We can work
with each resonance separately. The studied resonator is then
considered to be single mode.

A mode with an amplitude $a(t)$ is characterized by the
lifetime of the field τ_0 limited by losses, by the coupling
rate of the access line that is proportional to $1/\tau_e$ and by the
angular frequency of the resonance ω_0. The amplitude of the
input signal is denoted as $s_{in}(t)$; we can then calculate the
amplitude of the output signal $s_{out}(t) = -s_{in}(t) + \sqrt{2/\tau_e}\,a(t)$
by integration:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \left(\mathrm{j}\omega_0 - \frac{1}{\tau}\right) a(t) + \sqrt{\frac{2}{\tau_e}}s_{in}(t). \tag{1}$$

In equation (1), $\tau = (1/\tau_0 + 1/\tau_e)^{-1}$ is the lifetime of the
field in the cavity related to factor $Q = \omega_0\tau/2$. We assume
that the input signal is harmonic $s_{in}(t) = S_{in}\mathrm{e}^{\mathrm{j}\omega t}$ and the output
signal is $s_{out}(t) = S_{out}\mathrm{e}^{\mathrm{j}\omega t}$. As a consequence, the transfer
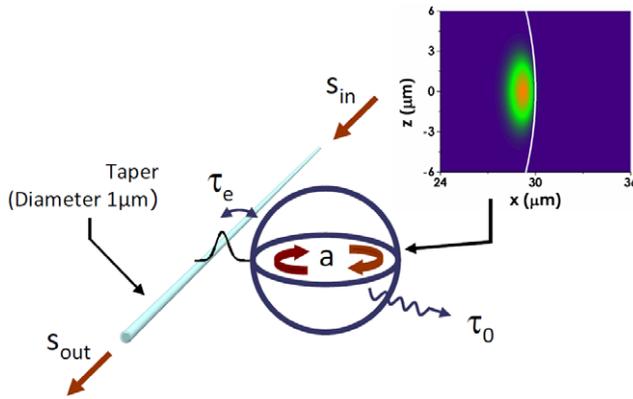
Figure 1. Schematic representation of the microresonator and its access line.

function is given by

$$T(\delta) = \left|\frac{s_{\text{out}}}{s_{\text{in}}}\right|^2 = \frac{(1/\tau_e - 1/\tau_0)^2 + 4\pi^2\delta^2}{(1/\tau_e + 1/\tau_0)^2 + 4\pi^2\delta^2}. \qquad (2)$$

In this expression δ is the frequency mismatch given by $\omega = \omega_0 + 2\pi\delta$. Different configurations are possible according to the relative value of τ_e and τ_0. If $\tau_0 \to \infty$ then the resonator has no losses, so $T(\delta) = 1$ as illustrated in figure 1(b). But if we suppose that $\tau_e = \tau_0$, then the resonator is in critical coupling and transmission at the resonance $T(0)$ becomes zero. If $\tau_0 < -\tau_e$, the resonator presents some gain (even if it stays less than the laser threshold) and it behaves like an amplifier; then $T(0) > 1$ [14].

3. Measurements

Amplification measurements were performed in erbium-doped ZBLALiP microspheres. Diameters of the microspheres are between 50 and 150 μm. These spheres are obtained by fusion in a microwave inductively coupled plasma [10]. The usual measurement method consists in sweeping the frequency of a laser emitting around 1550 nm, in order to obtain a description of one resonance for the microsphere. Simultaneously, this microsphere is pumped by a laser at 1480 nm propagating in the opposite direction, as described in figure 2.

Thanks to the pumping, the sphere delivers a laser signal for one or several resonances far from the studied resonance: it helps to stabilize the gain. An optical spectrum analyzer helps in controlling the microsphere laser emission. To illustrate how this technology can be applied, the laser frequency is swept slowly enough to stay in stationary approximation, to describe a narrow resonance as illustrated in figure 3.

Our approach is different. The analysis begins by rapidly sweeping the frequency of the laser. This process is analogous to what occurs when the speed is slow, but provides definitely an easier and unquestionably precise determination of the Q-factor. This approach is implemented by utilizing the following process. This allows us to work in the ringing

regime of the microsphere. It has already been demonstrated that it is possible to determine the characteristic parameters of the microsphere [11, 15] if we compare the profile of the ringing oscillations to an analytical model obtained thanks to equation (1) and to an incident field like $s_{\text{in}}(t) = S_{\text{in}}\text{e}^{\text{j}(\omega_i + \pi V_s t)t}$. The sweeping speed of the laser, V_S, is often difficult to obtain precisely. But it is directly obtained by the method described above. The results are illustrated in figure 4, where two time profiles are obtained on the same resonance, with two different speeds (the second one is two times faster) for sweeping the laser. While the deduced speeds are in a good ratio, adjustment provides almost the same results for τ_0 and τ_e. Observed differences for τ_0 and τ_e are mainly due to the fluctuations of the taper position. It has an influence on the signal coupling, and also on the pump. When performing the adjustment from the obtained ringing oscillation profiles, it has to be underlined that we deduce a quality factor as high as $Q = 5.8 \times 10^8$. The analysis proceeds by matching the theoretical and experimental curves. Q-factor estimation may be quickly obtained, and the efficiency of such a fast sweeping is really good. This is the desired result, which gives great insight into the resonator coupling and Q-factor estimation problem.

4. Coupling with a passive resonator and integration into an oscillator

An optoelectronic oscillator (OEO) with an optical resonator coupled to an optical fiber delivers a free spectral range microwave signal [1]. To implement this OEO in terms of miniaturization, it is possible to use a high WGM resonator with magnesium fluoride or calcium fluoride for microwave photonics applications. We have fabricated and characterized 5 mm diameter toroid MgF_2 and 12 mm CaF_2 resonators with a good surface roughness (1 nm). Quality factors better than 3.4×10^8 for magnesium fluoride [7] and 1.4×10^8 for calcium fluoride were measured by cavity ring-down measurements. Even if an optical line can be used as a frequency-selective component in the OEO loop, the use of the mini-resonator leads to a significant reduction of the oscillator size. Further work consists in measuring the phase noise on dedicated instruments developed at the laboratory [16, 17]. We can also reduce or eliminate sensitivity to vibration by using a refractive index adapter gel. The Q factor of resonators can still be improved by a better polishing of the crystal coupled with other treatments.

5. Conclusion

The work performed here is significant in the sense that it appears that the problem of the coupling between a high-Q-factor resonator and its external coupler has been theoretically and experimentally improved. The key is to recognize that oscillations of ringing induced by the sweeping of the excitation frequency of an active microsphere are observed. The measured apparent gain is approximately 27 dB (up to 33.9 dB in the stationary regime) in a 330 kHz band corresponding in the best case to a high quality factor
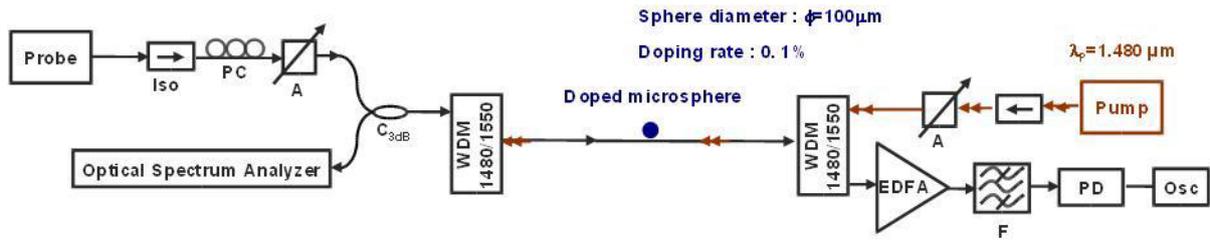
Figure 2. Setup for measuring the selective gain. Probe: extended cavity laser diode with a 150 kHz spectral width, emitting around 1550 nm. Iso: isolator, PC: polarization controller, C: 3dB coupler, WDM: multiplexer 1480/1550, PD: photodetector, EDFA: optical amplifier; F: tunable filter with a 50 pm spectral width, A: optical attenuator, Osc: oscilloscope. 'Pump' is a laser diode emitting around 1480 nm. The diameter of the microsphere is 105 μm.
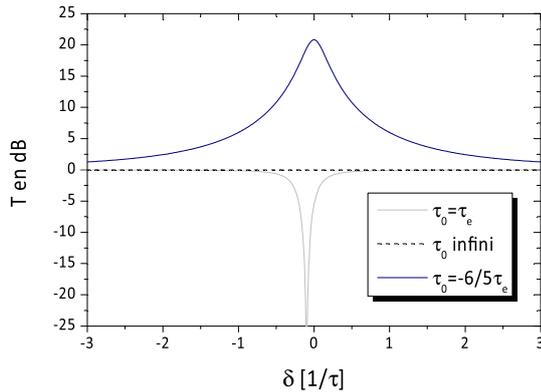

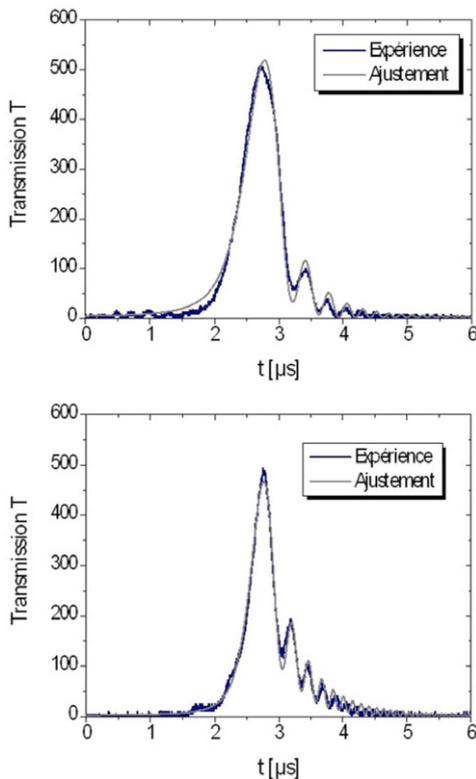**Figure 3.** Theoretical spectra obtained for $3\tau_0$.

Figure 4. Obtained ringing oscillation profiles for two frequencies F driving the sweeping of the tunable laser, PS = 125 nW, PP = 7 mW. Top: $F = 400$ Hz; these values are deduced from the adjustment: $\tau_0 = -40.9$ ns, $\tau_e = 38.8$ ns, VS = 2.2 MHz μs^{-1}, $Q = 4.4 \times 10^8$ and $T(0) = 31.2$ dB. Bottom: $F = 800$ Hz, also deduced from the adjustment: $\tau_0 = -39.2$ ns, $\tau_e = 37.6$ ns, VS = 4.1 MHz μs^{-1}, $Q = 5.8 \times 10^8$ and $T(0) = 33.9$ dB.

$Q = 5.8 \times 10^8$. All of these approaches have been developed to address high-quality factor resonator characterization. Addressing the combination of resonator polishing with other treatments, such as thermal annealing, to improve the surface roughness is still an active area of research to achieve higher Q factors.

Acknowledgment

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