# Estimation of the uncertainty for a phase noise optoelectronic metrology system

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#### Abstract

The configuration of the phase noise measurement system operating in the X-band (8.2–12.4 GHz) using a photonic delay line as a frequency discriminator is presented in this paper. This system does not need any excellent frequency reference and works for any frequency in this band. Oscillator frequency fluctuation is converted into phase frequency fluctuation through the delay line. The measured phase noise includes the device under test noise and the instrument background. Then the use of a cross correlation decreases the cross spectrum terms of uncommon phase noise as  $\sqrt{(1/m)}$ , where *m* is the average number. Using cross correlation on 500 averages, the noise floor of the instrument  $\pounds(f)$  becomes, respectively, -150 and -170 dBc Hz<sup>-1</sup> at 10<sup>1</sup> and 10<sup>4</sup> Hz from the 10 GHz carrier (-90 and -170 dBc Hz<sup>-1</sup> including 2 km delay lines). We then focus on determining the uncertainty. There are two categories of uncertainty terms: 'type A', statistic contributions such as repeatability and experimental standard deviation; 'type B' due to various components and temperature control. The elementary term of uncertainty for repeatability is found to be equal to 0.68 dB. Other elementary terms still have lower contributions. This leads to a global uncertainty of 1.58 dB at  $2\sigma$ .

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(Some figures may appear in colour only in the online journal)

# 1. Introduction

Recently, optoelectronic oscillators based on delay lines [1] or optics resonators [2–5] have been achieved. Their performances in terms of microwave frequency stability require a low phase noise system. A convenient method is to lock the oscillator to be measured on a reference standard. However, it is only possible to determine the phase noise for oscillators compared to another one with the same frequency if noise is expected to be better than a synthesizer. That is why there is a need to develop methods based on the use of two delay lines and cross correlation to be able to characterize an oscillator without referring to another one with the same delivered frequency [6]. We present in this short paper developments regarding an instrument dedicated to such a measurement for metrology applications and the evaluation of uncertainty.

# 2. Principle of the phase noise measurement system

A quasi-perfect RF-microwave sinusoidal signal can be written as

$$v(t) = V_0[1 + \alpha(t)]\cos(2\pi v_0 t + \varphi(t)),$$
(1)

where  $V_0$  is the amplitude,  $v_0$  the frequency,  $\alpha(t)$  the fractional amplitude fluctuation and  $\varphi(t)$  the phase fluctuation. Equation (1) defines  $\alpha(t)$  and  $\varphi(t)$  in low noise conditions:  $|\alpha(t)| \ll 1$  and  $|\varphi(t)| \ll 1$ . Short-term instabilities of signal are usually characterized in terms of the single-sideband noise power spectral density (PSD) S(f). *S* is typically expressed in units of decibels below the carrier per hertz (dBc Hz<sup>-1</sup>) and is defined as the ratio between the single-sideband noise power in 1 Hz bandwidth and the carrier power:

$$\pounds(f) = \frac{1}{2}S\varphi(f).$$
 (2)



Figure 1. The phase noise bench.

This definition given in equation (2) includes the effect of both amplitude and phase fluctuations. However, we must know the amplitude and phase noise separately because they act differently in the circuit. For example, the effect of amplitude noise can be reduced by an amplitude limiting mechanism and is mainly suppressed by using a saturated amplifier. Phase noise of microwave oscillators is usually characterized by heterodyne measurement. Whereas for such a system, we need a reference oscillator operating exactly at the frequency of the device under test (DUT) with lower phase noise, a reference oscillator is no longer required for homodyne measurement with a delay line discriminator. At microwave frequencies, electrical delay is not suitable because of its high losses. However, a photonic delay line offers high delay and low attenuation equal to  $0.2 \,\mathrm{dB \, km^{-1}}$ at a wavelength  $\lambda = 1.55 \,\mu$ m. The optoelectronic phase noise measurement system is schematically represented in figure 1. It consists of two equal and fully independent channels. The phase noise of the oscillator is determined by comparing the phase of the transmitted signal to a delayed replica through optical delay using a mixer. It converts the phase fluctuations into voltage fluctuations. An electro-optic modulator allows modulation of the optical carrier at microwave frequency. The length of the short branch where the microwave signal is propagating is negligible compared to the optical delay line. Mixers are used as phase detectors with both saturate inputs in order to reduce the amplitude noise contribution. Low pass filters are used to eliminate the high-frequency contribution of the mixer output signal. dc amplifiers are low flicker noise.

The oscillator frequency fluctuation is converted into phase frequency fluctuation through the delay line. If the mixer voltage gain coefficient is  $K_{\varphi}$  (V rad<sup>-1</sup>), then the mixer output rms voltage can be expressed as

$$V_{\text{out}}^2(f) = K_{\varphi}^2 |H_{\varphi}(\mathbf{j}f)|^2 S_{\varphi}(f), \qquad (3)$$

where  $|H_{\varphi}(jf)|^2 = 4\sin^2(\pi f\tau)$  is the transfer function of the optical delay line, and f is the offset frequency from the microwave carrier. Equation (3) shows that the sensitivity of the bench depends directly on  $K_{\varphi}^2$  and  $|H_{\varphi}(jf)|$ . The first is related to the mixer and the second essentially depends on the delay  $\tau$ . In practice, we need a fast Fourier transform (FFT) analyzer to measure the spectral density of noise amplitude  $V_{out}^2(f)/B$ , where B is the bandwidth used to calculate  $V_{out}(f)/B$ . The phase noise



**Figure 2.** Phase noise (dBc Hz<sup>-1</sup>) of the synthesizer measured at 10 GHz with  $K_{\varphi} = 425 \text{ mV rad}^{-1}$  and  $G_{\text{DC}} = 40 \text{ dB}$  versus the Fourier frequency.

of the DUT is finally defined by equation (4) and taking into account the gain of dc amplifier  $G_{DC}$  as

$$\pounds(f) = [V_{\text{out}}^2(f)] / [2K_{\varphi}^2 \cdot |H_{\varphi}(jf)|^2 G_{\text{DC}}^2 B].$$
(4)

#### 3. Validation of the performances

The measured phase noise includes the DUT noise and the instrument background. The cross correlation method allows us to decrease the cross spectrum terms of uncommon phase noise as  $\sqrt{(1/m)}$ , where *m* is the average number. Thereby uncorrelated noise is removed and the sensitivity of measure is improved. To validate the measure of our phase noise bench, we need to compare the datasheet of the commercial frequency synthesizer Anritsu/Wiltron 69000B [7] with the phase noise we measure using our system. Figure 2 shows the result of this measure. We can see that our bandwidth is limited to 100 kHz ( $\tau = 10 \,\mu s$ ) and the measured phase noise corresponds to the datasheet. Figure 3 presents the background phase noise of the bench after performing 500 averages with the cross correlation method on removal of the 2 km optical delay line. In this case, the phase noise of the 10 GHz synthesizer is rejected. The solid curve shows a noise floor (without the optical transfer function) better than -150 and -170 dBc Hz<sup>-1</sup> at  $10^1$  and  $10^4$  Hz from the 10 GHz carrier, respectively. The dotted curve is the noise floor when optical fiber is introduced.

#### 4. Evaluation of the uncertainty

There are two types of uncertainty terms [8]: the first type is called 'type A', statistical contributions such as reproducibility, repeatability, special considerations of fast Fourier transform analysis and the experimental standard deviation. The second type of uncertainty contributions is called 'type B' and is due to various components and temperature control like experience with or general knowledge of the behavior and properties of relevant materials and instruments, the manufacturer's specifications, the data provided in calibration and other certificates (noted BR)





**Figure 3.** The phase noise floor  $(dBc Hz^{-1})$  of the bench measured at 10 GHz with an Anritsu synthesizer (500 averages) versus the Fourier frequency.

and uncertainties assigned to reference data taken from handbooks. By the way, the uncertainty on  $\pounds(f)$  strongly depends on propagation of uncertainties through the transfer function as deduced from equation (4). Global uncertainty is strongly related to repeatability of the measurements. Repeatability is the variation in measurements obtained by one person on the same item and under the same conditions. Repeatability conditions include the same measurement procedure, the same observer, the same measuring instrument, used under the same conditions, repetition over a short period of time and the same location. We switch on all the components of the instrument and make a measurement keeping the data of the curve. Then we need to switch them off and switch them on again to obtain another curve. We must repeat this action several times until we have ten curves. The elementary term of uncertainty for repeatability is found to be equal to 0.68 dB. Other elementary terms still have lower contributions. For instance, temperature effects are less than 0.1 dB as the optical fiber regulation system of temperature is on. Resolution of instruments is determined by the value read on each voltmeter when we need to search for the minimum and maximum of the modulator but also for the wattmeter. Resolution is then no worse than 0.1 dB. Uncertainty in the determination of the coefficient  $K_{\varphi}$  dependent on the slope expressed in  $V \operatorname{rad}^{-1}$  is found to be less than 0.08 dB. For the contribution of the use of the automatic/manual range, we can deduce from the curves that this influence is not more than 0.02 dB. In our case, all these terms were found to be lower than repeatability. Other elementary terms have lower contributions. This leads to a global uncertainty of 1.58 dB. Table 1 summarizes each contribution of the elementary term of uncertainty.

### 5. Conclusions and further work

We detailed performances and consideration of the estimation of the uncertainty to show the main advantage of the instrument developed for metrology applications. With high performance better than  $-150 \,\text{dBc} \,\text{Hz}^{-1}$  at 10 Hz from the 10 GHz carrier, integration in an accredited laboratory is

Uncertainty	Designation	Value (dB)
A1	Repeatability	0.68
A2	Reproducibility	0
A3	Uncertainty term due to the number	
	of samples	0.1
А	$(\sum Ai^2)^{1/2}$	0.69
BR	Not applicable	0
BL1	Gain of the dc amplifier	0.04
BL2	Influence of the temperature	0.1
BL3	Influence of the resolution of the	
	instrument	0.1
BL4	Influence of the power of the DUT	0
BL5	Uncertainty on the determination of $K_{\varphi}$	0.08
BL6	Contribution of the automatic/	
	manual range	0.02
BL7	Influence of the variation of the input	
	power	0.02
BL	∑BLi	0.38
u <sub>C</sub>	$(A^2 + BR^2 + BL^2)^{1/2}$	0.79

Table 1. Budget of uncertainties.

expected and a new field of capabilities in terms of the calibration of a single source in the X-band will soon be achieved to extend the calibration metrology capabilities of the LNE (the French National Metrology Institute). It is interesting to underline that it is possible to determine the phase noise of a single oscillator in the X-band with a global uncertainty set to better than  $\pm 2 \text{ dB}$ .

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