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Uncertainty analysis for a phase-detector based phase noise measurement system

P. Salzenstein [a], T.Y. Wu [b,*]

[a] Centre National de la Recherche Scientifique (CNRS), Franche Comte Electronique Thermique Optique Sciences et Technologies (FEMTO-ST) Institute, 15 B, Avenue des Montboucons, F25030 Besancon, France

[b] National Metrology Centre, A*STAR, 1 Science Park Drive, Singapore 118221, Singapore

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ABSTRACT

Phase noise is an important parameter to characterise the frequency stability of oscillators and synthesised signal generators. Accurate measurement of phase noise is required for various applications in radar, communication and navigation systems. A single-channel phase-detector based phase noise measurement system is described. The system's measurement errors and uncertainties have been analysed in details. The expanded uncertainty is about 2.7 dB for calibrating phase noise of a signal generator at 0.001–1.6 GHz for frequency offsets from 1 Hz to 100 kHz. The uncertainty budget for measuring a signal generator's phase noise at 640 MHz is also presented.

1. Introduction

Phase noise is used to characterise the frequency stability of oscillators and synthesised signal generators [1–3]. Radar systems, communications systems, precision navigation and many other areas require oscillators with low phase noise. Phase noise of frequency synthesisers will reduce receiver sensitivity in transmission measurement, microwave imaging and wireless communication system [4–8]. It is important to measure the phase noise with high accuracy to ensure system performance. It has been a complicated task for an electrical engineer to measure phase noise. It is also important to understand the various uncertainty components in phase noise measurement. An accuracy and uncertainty analysis had been given in [9] for a single-channel phase-detector based system specially built for phase noise measurement (with low-phase-noise frequency reference sources). Their uncertainty analysis had been conducted a few years before the Guide to the expression of Uncertainty in Measurement (GUM) [10] was published, therefore their approach is different from the GUM. A GUM-based measurement uncertainty analysis of a dual-channel cross-correlation based phase noise measurement system has been presented in [11,12]. This paper presents the measurement uncertainty analysis of a single-channel phase-detector based phase noise measurement system using an approach recommended by the GUM [10]. This system is based on a commercial instrument (Agilent E5504B) equipped with a frequency synthesiser as the reference source. The uncertainty budget for measuring a signal generator's phase noise at 640 MHz is also presented.

2. Phase noise measurement

The phase noise of the device under test (DUT) can be expressed in terms of $S_\phi(f)$, the single-sided spectral density of mean-squared phase fluctuations per 1 Hz bandwidth, given by

* Corresponding author.
E-mail addresses: patrice.salzenstein@femto-st.fr (P. Salzenstein), thomas_wu@nmc.a-star.edu.sg (T.Y. Wu).

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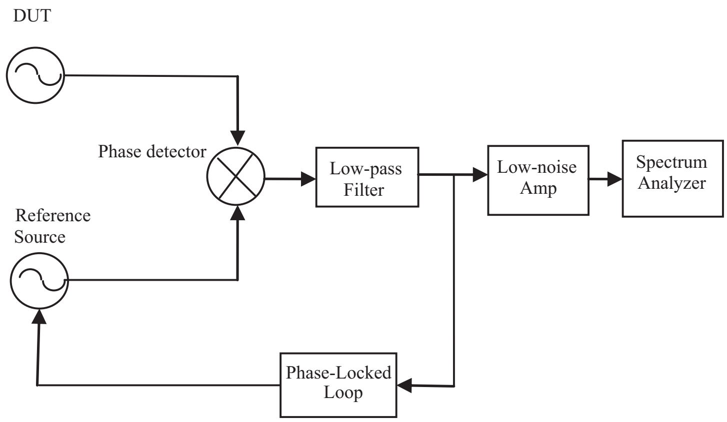

Fig. 1. Block diagram of a single-channel phase-detector based phase noise measurement system.

$$S_\phi(f) = \frac{\delta\phi^2_{rms}(f)}{BW} \quad (\mathrm{rad}^2/\mathrm{Hz}) \tag{1}$$

where $\delta\phi^2_{rms}(f)$ is the mean-squared phase fluctuations, measured at a frequency offset of f from the carrier, in a bandwidth of BW. When the total phase deviations are much less than 1 radian, the single-sided phase noise of the DUT can be derived [13] as

$$L(f) = 10\log\frac{S_\phi(f)}{2} \quad (\mathrm{dBc/Hz}) \tag{2}$$

Typical phase noise measurement techniques include direct spectrum method, phase detector technique, frequency discriminator method and dual-channel cross-correlation technique [13–15]. In this paper, we studied a 0.001–1.6 GHz phase noise measurement system which is based on a commercial instrument (Agilent E5504B) using single-channel phase detector technique as illustrated by Fig. 1.

Any phase fluctuation will lead to voltage fluctuation at the mixer output. When the phase deviation is much less than 1 radian, the output voltage is considered linear with the phase changes [9].

$$V_{rms}(f) = K_\phi \delta\phi_{rms}(f) \tag{3}$$

where K_ϕ is the phase conversion factor. This phase to voltage conversion can be used to estimate the power spectral density of the phase fluctuations $S_\phi(f)$. The phase noise $L(f)$ can be obtained as

$$L(f) = 10\log\frac{V^2_{rms}(f)}{2K^2_\phi BW} \quad (\mathrm{dBc/Hz}) \tag{4}$$

3. Evaluation of measurement uncertainties

The errors in measured phase noise of a DUT (in dBc/Hz) using phase detector method include the following error components: error due to system noise floor (E_s), error due to reference source's phase noise (E_r), error due to phase conversion factor (E_k), error due to temperature variation (E_t), error due to spectrum analyser's (SA) frequency response (E_{fr}), SA's linearity (E_L), SA's resolution bandwidth switching (E_{rb}) and relative amplitude measurement error (E_{ra}), and measurement repeatability. The standard uncertainty for each error component has been analysed according to the GUM [10]. The uncertainties are not just 'typical' uncertainties for each error component. They have been derived based on the corresponding error probability distribution and the error bound. The details of the type B and type A uncertainty component evaluation are given in the following sections.

3.1. Error due to reference source phase noise

The phase noise power (in watt) measured by the phase detector is actually the sum of the phase noise power of DUT and the phase noise power of the reference source. A microwave signal generator with low phase-noise option (Agilent E8257D) is used as the reference source at 0.001–1.6 GHz in the system for measuring a DUT with phase noise level higher than that of the reference source. At a carrier frequency of 640 MHz, the phase noise of the reference source has been measured by the Agilent E5504B system using a 640 MHz oscillator with ultra-low phase noise. The phase noise power of reference source (P_r, in watt) is found to be at least 2 dB below that of DUT (P_d, in watt) for offset range from 1 Hz to 100 kHz $\left(10\log\frac{P_d}{P_r} = 2\ dB\right)$. Thus the maximum error in DUT phase noise measurement (for carrier at 640 MHz) due to the reference source can be derived as:

$$e_r = 10\log\frac{P_r + P_d}{P_d} = 10\log\left(1 + \frac{P_r}{P_d}\right) = 2.12\ \mathrm{dB} \tag{5}$$

The error due to reference source can be assumed to have rectangular distribution. Thus the standard uncertainty of this contribution for DUT phase noise measurement is derived to be $u_r = e_r/\sqrt{3} = 1.23$ dB [10].

3.2. Error due to system noise floor

When measuring a DUT's signal carrier at 640 MHz (power level is 15 dBm), the Agilent E5504B system noise floor is measured to be −178 dBc/Hz at 100 kHz offset by the E5504B system, which is 39.4 dB below the DUT phase noise level at 100 kHz frequency offset. DUT phase noise at 1 Hz to 10 kHz offset is over 40 dB higher than the system noise floor, hence the maximum error due to Agilent E5504B's system noise floor can be obtained by calculating the contribution of system noise floor to DUT phase noise at 100 kHz offset, as given by

$$e_s = 10\log\frac{P_s + P'_d}{P'_d} = 10\log\left(1 + \frac{P_s}{P'_d}\right)\ \text{dB} \tag{6}$$

where P_s (in watt) refers to the phase noise power due to system noise floor, P'_d (in watt) refers to the DUT phase noise power for frequency offset at 100 kHz and $10\log\frac{P'_d}{P_s} = 39.4$ dB. The maximum error e_s is derived to be 0.0005 dB for the offset range from 1 Hz to 100 kHz. The error due to system noise floor is assumed to have rectangular distribution, and its standard uncertainty is calculated to be $u_s = 0.0005/\sqrt{3} = 0.0003$ dB [10].

3.3. Errors due to phase conversion factor

The Agilent E5504B's phase noise measurement software calibrates the phase conversion factor (K_ϕ) by generating a low-frequency beat note signal at phase detector output before the DUT phase noise is measured. The K_ϕ is obtained by measuring the beat note signal's slope around the positive and negative zero-crossing points.

$$K_\phi = \frac{\Delta V}{\Delta\phi G} \tag{7}$$

where G is the gain of the low-noise amplifier and

$$\Delta\phi = \frac{2\pi(t_2 - t_1)}{T} \tag{8}$$

where t_1 is the time when beat note signal is sampled at zero Volt, t_2 is the time when beat note signal is sampled at ΔV (its amplitude value), $t_2 = 0.032$ ms, T is the period of the beat note signal ($T = 0.001$ s) and $\Delta\phi \ll 1$ radian. We can study K_ϕ in dB scale,

$$20\log K_\phi = 20\log\Delta V + 20\log T - 20\log G - 20\log(t_2 - t_1) - 20\log(2\pi) \tag{9}$$

since $t_1 \ll t_2$, we have

$$20\log K_\phi \approx 20\log\Delta V + 20\log T - 20\log G - 20\log t_2 - 20\log(2\pi) \tag{10}$$

The data acquisition card sampling interval $T_s = 0.1$microseconds. There is uncertainty of 0.1 microseconds in measurement of t_2 due to the discrete sampling in time. The maximum error in K_ϕ caused by error in t_2 can be derived as

$$e_{ts} = 20\log\frac{t_2 + T_s}{t_2} = 0.027\ \text{dB} \tag{11}$$

According to the instrument specification, the maximum error in measurement of ΔV is $e_v = 0.11\% \cdot \Delta V$. thus the maximum error in K_ϕ caused by error in measuring ΔV can be derived as

$$e_{dv} = 20\log\frac{\Delta V + e_v}{\Delta V} = 20\log(1 + 0.0011) = 0.01\ \text{dB} \tag{12}$$

There is a maximum relative error of 1% in determination of the amplifier's gain according to its specification. The maximum error in K_ϕ due to error in G is derived as

$$e_g = 20\log\frac{G + G\cdot 1\%}{G} = 0.086\ \text{dB} \tag{13}$$

The maximum error due to uncertainty in T is derived as

$$e_{tt} = 20\log\frac{T + T_s}{T} \tag{14}$$

Since $T \gg T_s, e_{tt}$ is negligible.

Thus the maximum error in determination of K_ϕ can be calculated from

$$e_k = \sqrt{e_{ts}^2 + e_{dv}^2 + e_g^2} = 0.091\ \text{dB} \tag{15}$$

The standard uncertainty in phase noise measurement due to K_ϕ is derived as $u_k = e_k/\sqrt{3} = 0.053$ dB, assuming this error component has a rectangular distribution.

3.4. Errors due to spectrum analyser amplitude measurement accuracy

As shown in (4), the error in measurement of $V^2_{rms}(f)$ by the spectrum analyser (SA) will affect the phase noise measurement accuracy. We should consider the high dynamic range and variations in power amplitude measurement by the SA (for various frequency offsets). According to the SA's specification, the maximum error in SA relative amplitude measurement is $e_{ra} = 0.7$ dB, maximum error in SA resolution bandwidth switching is $e_{rb} = 0.3$ dB, and maximum error in SA frequency response is $e_{fr} = 0.5$ dB.

The SA linearity error has been checked by measuring a 0–10 dB variable attenuator's incremental attenuation using the SA at 1 MHz. The variable attenuator is connected between a signal generator and the SA. The signal generator's output power level is decreased from 10 dBm to −80 dBm in 10 dB step at 1 MHz when the variable attenuator's incremental attenuation is measured by the SA. The difference between the measured incremental attenuation and its calibrated attenuation value reveals the linearity error of the SA. The maximum error in SA's linearity is found to be $e_L = 0.085$ dB. Variation of linearity error for other frequency offset from 1 Hz to 1 MHz is covered by the frequency response error.

Theses four error components due to SA amplitude measurement accuracy can be assumed to have rectangular distributions and their standard uncertainty values ($u_{fr}, u_L, u_{rb}, u_{ra}$) are obtained by dividing their maximum errors by $\sqrt{3}$ respectively (as shown in Table 1).

3.5. Error due to temperature variation

The environment of the laboratory is in a controlled range. The laboratory's temperature is 23 ± 2 degrees and relative humidity is 55%± 10%. The phase noise of oscillator will increase by $10\log(T_2/T_1)$ when its temperature increases from T_1 to T_2 (in Kelvin) according to Leeson's equation [1] which states that the phase noise power (in watt) is proportional to the absolute temperature. The maximum change in phase noise of DUT due to

Table 1
Measurement uncertainty budget for measuring phase noise of a signal generator at 640 MHz (for frequency offset from 1 Hz to 100 kHz).

Uncertainty components	Type	Probability distribution	Values (dB)
System noise floor (u_s)	B	Rectangular	0.0003
Reference source phase noise (u_r)	B	Rectangular	1.23
Phase conversion factor (u_k)	B	Rectangular	0.053
SA frequency response error (u_{fr})	B	Rectangular	0.029
SA linearity error (u_L)	B	Rectangular	0.050
SA resolution bandwidth switching (u_{rb})	B	Rectangular	0.17
SA relative amplitude error (u_{ra})	B	Rectangular	0.40
Temperature variation (u_t)	B	Rectangular	0.034
Meaurement repeatability (u_a)	A	Normal	0.30
Combined standard uncertainty			1.37
Expanded uncertainty ($k = 2$)			2.7

temperature variation is $e_1 = 10\log(298/296) = 0.0292$dB. Similarly, the maximum change in phase noise of reference source due to temperature variation is $e_2 = 10\log(298/296) = 0.0292$dB. Since the measured phase noise power is the sum of the DUT phase noise power and the reference source's phase noise power, the temperature variation will generate a maximum error (in the measured phase noise) given by

$$e_t = e_1 + e_2 = 0.0584 \text{ dB} \tag{16}$$

This random error has a rectangular distribution, therefore the standard uncertainty due to temperature variation is derived as $u_t = e_t/\sqrt{3} = 0.034$ dB.

3.6. Repeatability

The phase noise measurement is repeated 4 times for each carrier frequency. Frequency offset is from 1 Hz to 100 kHz. The standard uncertainty due to measurement repeatability error (type A uncertainty) [10] is obtained as

$$u_a = \sqrt{\frac{\sum_{k=1}^{n}(x_k - \bar{x})^2}{n(n-1)}} \tag{17}$$

where

$$\bar{x} = \frac{\sum_{k=1}^{n} x_k}{n} \tag{18}$$

and x_k are the phase noise measurement data for each offset frequency and n is the number of measurement ($n = 4$). The type A uncertainty is assumed to be the uncertainty on the mean value ($\bar{x}$).

3.7. Combined and expanded uncertainty

The various type B uncertainty components and the measurement repeatability are combined using root-sum-of-the-squares method to yield the standard system uncertainty, u_s, given by

$$u_s = \sqrt{u_s^2 + u_r^2 + u_k^2 + u_{fr}^2 + u_L^2 + u_{rb}^2 + u_{ra}^2 + u_t^2 + u_a^2} \tag{19}$$

The expanded system uncertainty is

$$U_s = ku_s \tag{20}$$

where k is the coverage factor and $k = 2$ [10]. The expanded uncertainty U_s defines an interval having a level of confidence of approximately 95%. Table 1 gives the measurement uncertainty budget for measuring phase noise of a microwave signal generator at 640 MHz (for frequency offset from 1 Hz to 100 kHz). The measurement repeatability is 0.3 dB. The uncertainty component due to reference source's phase noise is the dominating term. The phase noise expanded measurement uncertainty is 2.7 dB for frequency offset from 1 Hz to 100 kHz.

4. Measurement results

Phase noise of a signal generator (R&S SMF100A) has been measured by an Agilent E5504B system at the carrier frequency of 640 MHz for the offset from 1 Hz to 100 kHz. To validate the uncertainty evaluation for the phase noise measurement made by E5504B using single-channel phase detector method, the same signal generator has been measured by an Agilent E5052B phase noise measurement system at carrier frequency of 640 MHz. In the E5052B system, dual-channel cross correlation method has been used (with correlation number = 100) to reduce the contribution from phase noise of its reference source. The measurement uncertainty of the E5052B is 2.0 dB according to its specifications. The phase noises (mean value) of SMF100A measured by Agilent E5504B and Agilent E5052B system (at a carrier frequency of 640 MHz) have been plotted in Fig. 2.

Table 2 shows the phase noises (mean value) of DUT (SMF100A) measured by Agilent E5504B and E5052B system, their measurement expanded uncertainties, difference in measurements and the normalised error (E_n). The E_n is defined as

$$E_n = \frac{L_1 - L_2}{\sqrt{U_1^2 + U_2^2}} \tag{21}$$

where L_1 is the phase noise of DUT measured by E5504B with expanded measurement uncertainty of U_1 ($U_1 = 2.7$ dB), L_2 is the phase noise of DUT measured by E5052B with expanded measurement uncertainty of U_2 ($U_2 = 2.0$ dB). Table 2 shows that $E_n < 1$ for all measurements, thus it can be concluded that the two measured values for each offset frequency agree with each other within their expanded uncertainties.

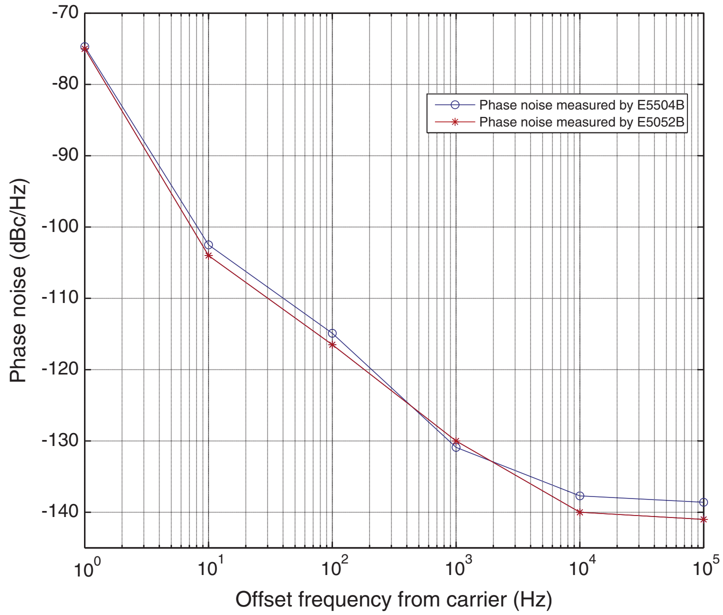

Fig. 2. The phase noise (mean value) of SMF100A measured by Agilent E5504B and E5052B system at a carrier frequency of 640 MHz.

Table 2
Comparison of phase noise of a DUT (SMF100A) measured by Agilent E5504B and E5052B system at a carrier frequency of 640 MHz.

Offset frequency (Hz)	Phase noise measured by E5504B (L_1, dBc/Hz)	Expanded uncertainty of E5504B system (U_1, dB)	Phase noise measured by E5052B (L_2, dBc/Hz)	Expanded uncertainty of E5052B (U_2, dB)	$L_1 - L_2$ (dB)	E_n
1	−74.7	2.7	−75.0	2.0	0.3	0.10
10	−102.5	2.7	−104.0	2.0	1.5	0.44
100	−114.9	2.7	−116.5	2.0	1.6	0.47
1000	−130.9	2.7	−130.0	2.0	0.9	−0.28
10,000	−137.7	2.7	−140.0	2.0	2.3	0.68
100,000	−138.6	2.7	−141.0	2.0	2.4	0.72

5. Conclusion

We have analysed measurement errors in a single-channel phase-detector based phase noise measurement system at 0.001–1.6 GHz. The analysis has considered the measurement errors caused by repeatability, reference source phase noise, system noise floor, phase conversion factor, temperature variation, SA's frequency response, SA's linearity, SA's resolution bandwidth switching and its relative amplitude measurement error. The expanded uncertainty is about 2.7 dB for calibrating phase noise of a signal generator at 0.001–1.6 GHz for frequency offset from 1 Hz to 100 kHz. The advantage of a single-channel phase-detector based phase noise measurement system is its low cost and high speed in measurement, although its uncertainty is slightly higher than that of a dual-channel cross-correlation phase noise measurement system. The measurement uncertainty can be further reduced if a frequency reference source with phase noise much lower than the DUT is used in the measurement.

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