Applications using complexity of electro-optic delay dynamics: from chaos to fixed points through limit cycles

L. Larger[1]

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Conference on Delayed Complex systems

towards a PHOtonic liquid state machine based on delay-CoUpled Systems

Outline

1. Introduction: where does it come from
   - From Ikeda ring cavity to OE NL delayed feedback
   - Modeling point of views
2. Cross-fertilization between fundamental and applications
   - Optical chaos communications
   - High spectral purity microwave limit cycle in OEO
   - Photonic Neuromorphic computing from the steady state
3. Conclusion: even more is remaining

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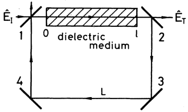
Ikeda dynamics: a bit of history

From an Optics Gedanken experiment to a flexible photonic system



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  (Ikeda, *Opt.Commun. 1979*).

- Bulk electro-optic setup

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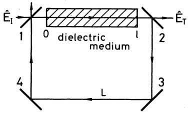

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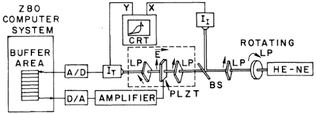

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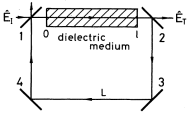

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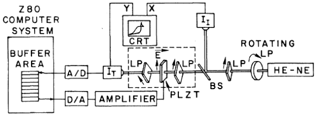

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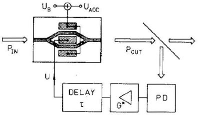

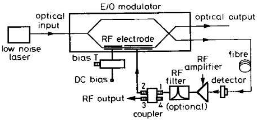

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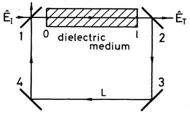

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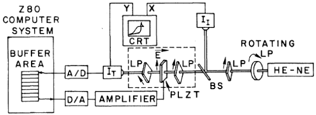

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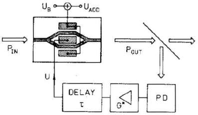

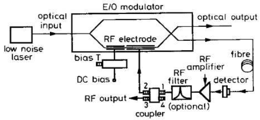

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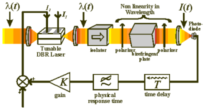

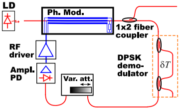

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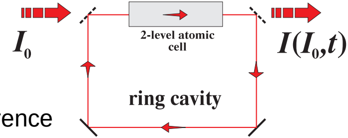
Physics modeling: all-optical ring cavity

- **Experimental setup**
  - Response time of the Kerr medium
  - Delay $\equiv$ Cavity round trip
  - NL function $\equiv$ Input & Feedback interference
  - Interf. condition ruled by Kerr phase shift



- **Model**

$$\frac{d\varphi}{dt} = -\omega_{n_2}\varphi(t) + \gamma|E(t)|^2$$
$$|E(t)|^2 = A^2\{1 + 2B\cos[\varphi(t-\tau_R) - \varphi_0]\}$$

- **Main Features**
  - dynamics $\equiv$ linear filtering, dielectric medium response
  - scalar DDE, single variable ($\varphi(t)$, or $I(t) \propto |E(t)|^2 = F_{NL}[\varphi(t-\tau_D)]$)
  - potentially ultra-fast, interferometric mechanical stability
  - relatively low feedback gain (Kerr efficiency)

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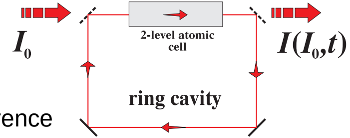

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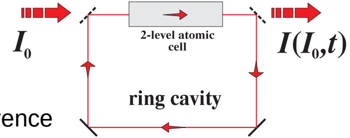

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"Signal processing" modeling (system approach)

Nonlinear delayed feedback oscillator

- **Experimental setup**
  - Differential process ≡ linear filter: $H(\omega) = \mathrm{FT}[h(t)]$
  - Delay ≡ propagation time of a carrier wave travelling at velocity v
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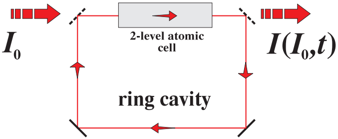

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$$\tau \frac{\mathrm{d}x}{\mathrm{d}t} + x(t) = F_{\mathrm{NL}}[x(t-\tau_D)] \; (= \beta \cos^2[x(t-\tau_D) + \Phi_0])$$

$$x(t) = \int_{t_0}^{t} h(t-\xi)\, F_{\mathrm{NL}}[x(\xi - \tau_D)\mathrm{d}\xi]$$

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  - dynamics ≡ linear (electronic, thus slower) filtering feedback
  - (event.) higher order DDE
  - qualified Telecom devices: stable, reliable, reconfigurable
  - (potent.) high gain (drive voltage vs. half wave -EO- voltage)

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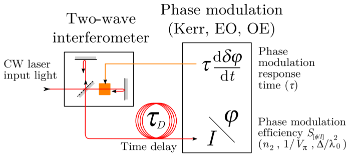

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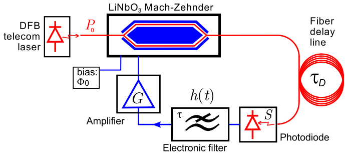

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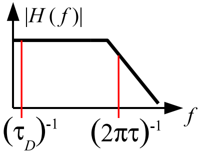
Ikeda-like dynamics: 1st order low pass filter

- **Differential equation derivation.**
  - Fourier and time domains correspondance $(\mathrm{d}/(\mathrm{d}t) \leftrightarrow \times i2\pi f)$
  - Linear filter described by polynomial fractional



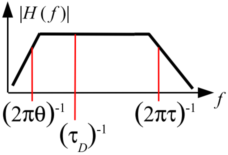
- **Low pass dynamics**
  - Differential process: 1^{st} order low pass filter
  - 2-time scales only (typ. large delay case $\tau_D \gg \tau$, for Ikeda instabilities, period doubling)

$$\tau \frac{\mathrm{d}x}{\mathrm{d}t}(t) + x(t) = z(t) \qquad \leftrightarrow \qquad H(f) = \mathrm{FT}[h(t)] = \frac{X(f)}{Z(f)} = \frac{1}{1 + i2\pi f \tau}$$

- $z(t) = \beta \cos^2[x(t-\tau_D) + \Phi]$, NL delayed self-feedback driving force

integro-Differential Delay Equation: bandpass case

- i **DDE:** the NL delay damped oscillator viewpoint
  - Simplest polynomial fractional for a bandpass filter: 2nd order (strongly damped harmonic oscillator $m \gg 1$, $\theta = \frac{2m}{\Omega_0}$, $\tau = \frac{1}{2m\Omega_0}$)
  - Higher orders sometimes important (2nd order usually enough qualitatively)



$$H(\omega = 2\pi f) = \frac{X(\omega)}{Z(\omega)} = \frac{i\frac{2m}{\Omega_0}\omega}{1 + i\frac{2m}{\Omega_0}\omega - \frac{\omega^2}{\Omega_0^2}} \quad \leftrightarrow \quad \frac{1}{\Omega_0^2}\frac{\mathrm{d}^2x}{\mathrm{d}t^2}(t) + \frac{2m}{\Omega_0}\frac{\mathrm{d}x}{\mathrm{d}t}(t) + x(t) = \frac{2m}{\Omega_0}\frac{\mathrm{d}z}{\mathrm{d}t}(t)$$

or $$\frac{1}{\theta}\int_{t_0}^{t} x(\xi)\,\mathrm{d}\xi + x(t) + \tau\frac{\mathrm{d}x}{\mathrm{d}t}(t) = z(t) - z(t_0)$$

- $z(t) = \beta\, F_{\mathrm{NL}}[x, t, t - \tau_D]$ NL delayed self-feedback driving force

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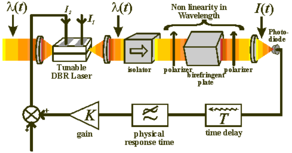

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- **Record non linearity strength** up to 14 extrema
- **FM chaos:** operating principles transfered to electronics

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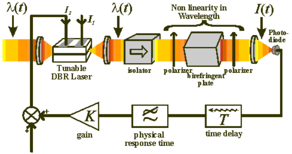

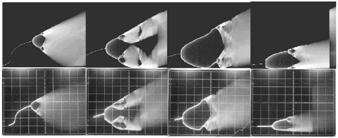

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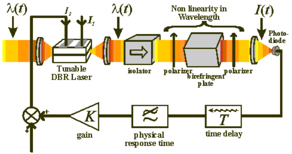

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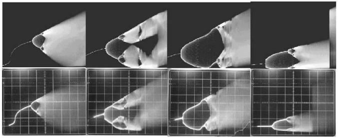

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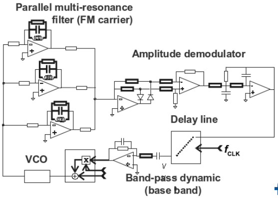

Fundamental issue: Sub-critical Hopf bif. in DDE?

Optical chaos communication demonstrated, but also . . . :
(EU Patent 96, Goedgebuer *et al.*, Phys. Rev. Lett. 98)

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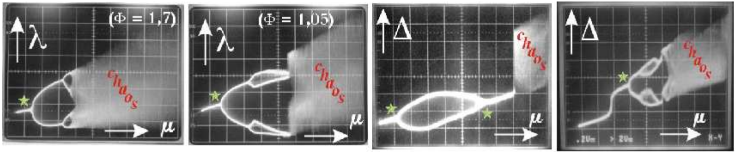

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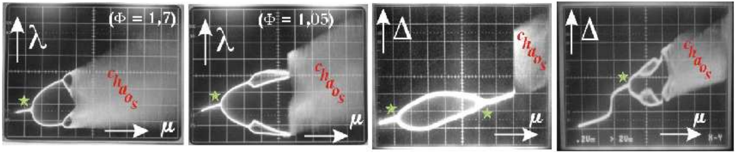

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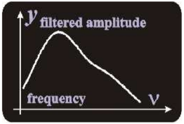

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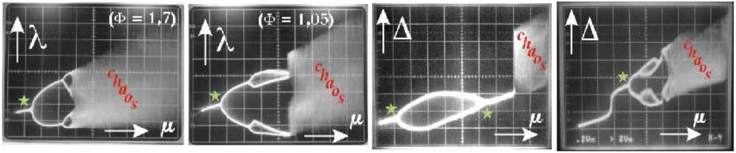

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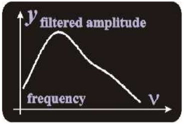

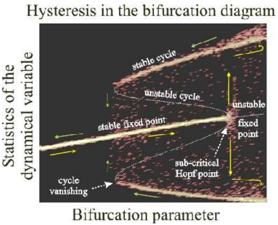

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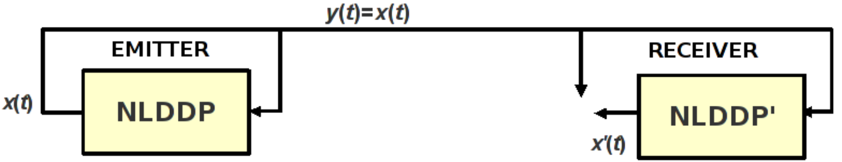
Uni-directional (Master-Slave) chaos commnications:

Self synchronization (replication), chaos masking & unmasking

- **Convolution product description**
- **Intensity EO delay dynamics (broadband, bandpass)**



$$x(t)=\{h_\theta * f_{NL}(y)_{\theta-T}\}(t) \qquad x'(t)=\{h'_\theta * f'_{NL}(y)_{\theta-T'}\}(t)$$

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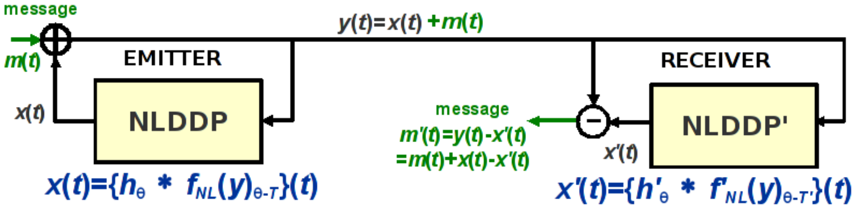

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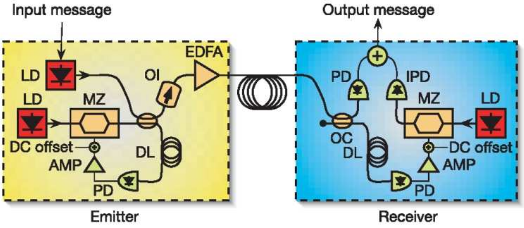
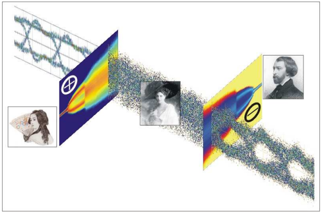
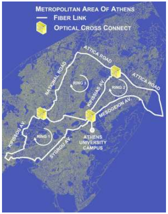


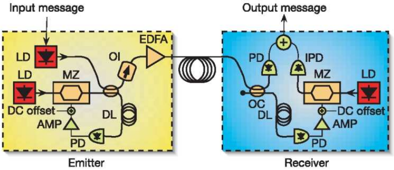
EO intensity chaos communications results

- **Intensity chaos "spy suitcases".**





- **Field experiment during OCCULT @ 3Gb/s.**



Vol 437|17 November 2005|doi:10.1038/nature04275 nature

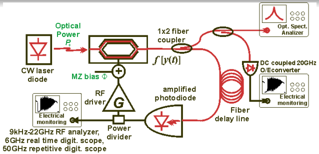
LETTERS

Chaos-based communications at high bit rates using commercial fibre-optic links

Apostolos Argyris[1], Dimitris Syvridis[1], Laurent Larger[2], Valerio Annovazzi-Lodi[3], Pere Colet[4], Ingo Fischer[5]†, Jordi García-Ojalvo[6], Claudio R. Mirasso[7], Luis Pesquera[8] & K. Alan Shore[9]

Also fundamental nonlinear dynamics issues

- **Bifurcation parameter: Φ.**



- **Dynamics at fixed β.**

(Kouomou *et al.*, *Phys. Rev. Lett. 2005*, Peil *et al.*, *Phys. Rev. E 2009*,T.E. Murphy *et al.*, *Phil. Trans. R. Soc. A, 2010*, Callan *et al. Phys. Rev. Lett. 2010*, Larger & Dudley, *Nature*, News & Views, 2010).

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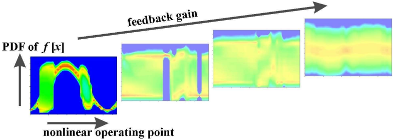
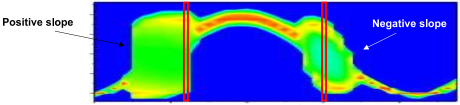
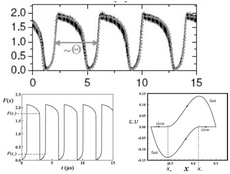
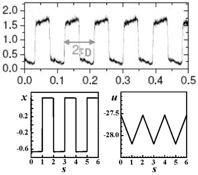




$$x' = z,$$
$$\tau z' = -z - \theta^{-1}x + \beta\left[\cos^2\left(x(t-\tau_D)+\Phi\right)\right]'$$

$$y' = x,$$
$$\tau x' = -x - \theta^{-1}y + \beta\left[\cos^2\left(x(t-\tau_D)+\Phi\right) - \cos^2(\Phi)\right]$$





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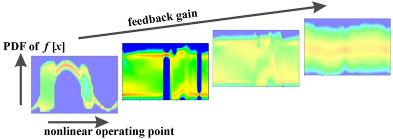

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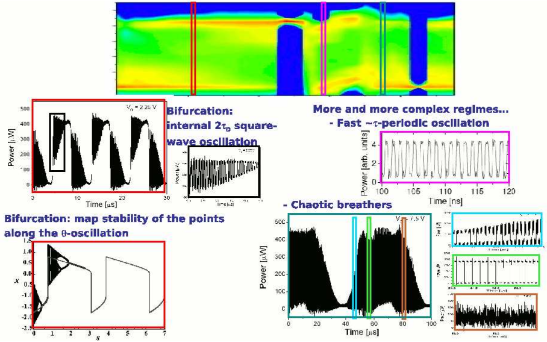

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Further setup evolutions motivated by applications

Electro-optic phase delay dynamics

- **Setup, physical principles.**
  - DPSK optical modulation
  - Temporally nonlocal non linearity
  - Intrinsically high speed
- **ΦM in the optical spectrum.**

(Lavrov *et al.*, *Phys. Rev. E 2009*).

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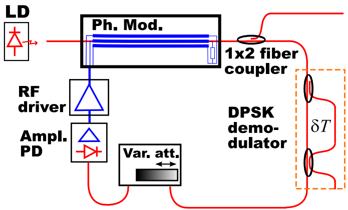

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(Lavrov *et al.*, *Phys. Rev. E 2009*).

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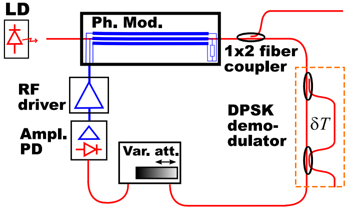

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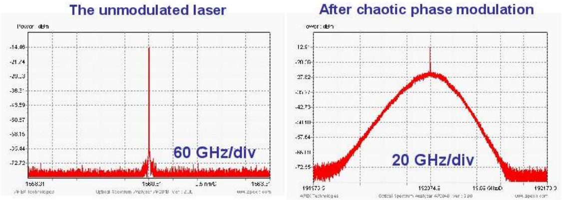
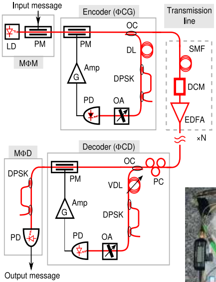
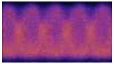


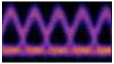
(Lavrov *et al.*, *Phys. Rev. E 2009*).

Field experiment @ 10 Gb/s

Emitter setup packaged on a A4-alumni board

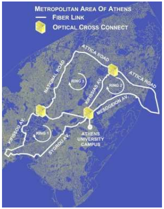






Receiver setup packaged on a A4-alumni board

“Lumière” brothers ring network in Besançon, France (22km)



Athens, Greece, metropolitan fiber network (116km)

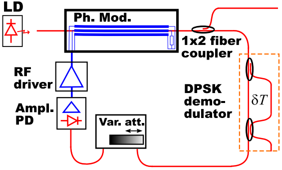
(Lavrov *et al.*, *IEEE J.Quant.Electron. 2010*).

Modeling

- **The dynamics**
  - Integro-differential (linear bandpass filter) nonlinear delay equation

$$\frac{1}{\theta}\int_{t_0}^{t}\varphi(\xi)\,\mathrm{d}\xi+\varphi(t)+\tau\frac{\mathrm{d}\varphi}{\mathrm{d}t}(t)=\beta\cdot\left[f_{(t-\tau_D)}(\varphi^\star)\right]$$

  - Non linearity via imbalanced interferometer (temporal non locality)



- standard DPSK demodulator

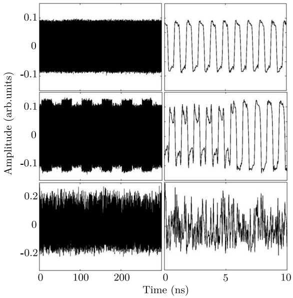
$$f_t(\varphi)=\{1+\cos[\varphi(t)-\varphi(t-\delta T)+\Phi_0]\}$$

- generalized multiple wave interferometer

$$f_t(\varphi)=F_0\left|1+\sum_k\alpha_k\,e^{i[\varphi(t)-\varphi(t-\delta T_k)+\Phi_k]}\right|^2$$

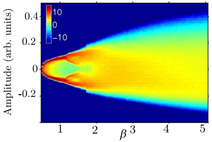
Dynamical issues: crenelated envelop bifurcations

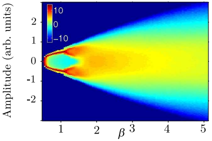
- **Φ-chaos: 4 time-scales ($\theta \gg \tau_D \gg \delta T \gg \tau$)**



Temporal bif. diagrams →

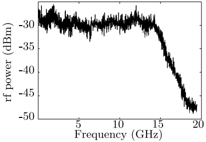
← Time traces

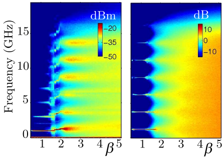




Spectral bif. diagram →

← Flat chaotic rf spectrum



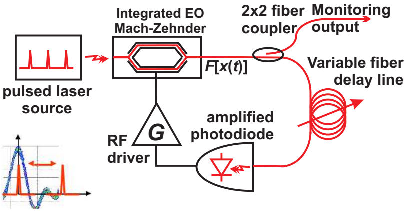


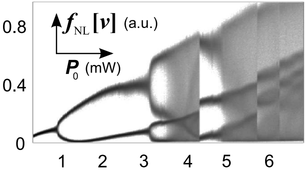
(Lavrov *et al.*, *Phys. Rev. E 2009; Weicker* et al., *Phys.Rev. E 2012*).

Chaos comm. & data packet compatibility. . .

An EO delay dynamics performing a Map?

- **Continuous amplitude, but discrete time dynamics:** required for packet routing of clocked Telecom data
- **Seed the EO setup with a pulsed laser**
- **Experimental Bifurcation diagram with clear period-3 windows**
- **Dimension** $D_N \equiv$ **pulses / delay**
- D_N **independent maps**





(Larger *et al. Phys. Rev. Lett.* 2005, Grapinet *et al.*, *Chaos* 2008; Grapinet *et al.*, *Electron. Lett.* 2008)

Outline

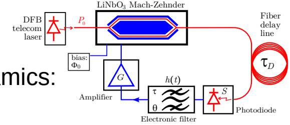
1. Introduction: where does it come from
2. Cross-fertilization between fundamental and applications
   - Optical chaos communications
   - High spectral purity microwave limit cycle in OEO
   - Photonic Neuromorphic computing from the steady state
3. Conclusion: even more is remaining

A brief OEO history, and principles

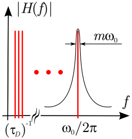
Large delay resonant (narrow band) dynamics

- Common history with Ikeda setups, at the origines
  (Neyer and Voges, *Appl. Phys. Lett. A, 1981*).
- Stochastic description of delay-induced spectral purity
  (Pomeau, *C.R. Acad. Sc. Paris 1986*).
- Time-Frequency metrology interest in the US (NASA, JPL)
  (*Yao and Maleki, Electron.Lett., 1994*).
- Start-Up company *OEwaves* created in 2000
- Unconventional optoelectronic approach in TF, only recently recognized as a high potential approach (together with high Q-factor optical resonators)
  (Fortier *et al., Nature Photon., 2011*).
- Principle summary: *large delay* $\equiv$ *high energy storage time*

i DDE modeling for the microwave delay oscillator



- **Bifurcation parameters**



  - Narrow band ($m \ll 1$) nonlinear delay dynamics:

$$\frac{2m}{\Omega_0}\int_{t_0}^{t} x(\xi)\,\mathrm{d}\xi + x(t) + \frac{1}{2m\Omega_0}\frac{\mathrm{d}x}{\mathrm{d}t}(t) = \beta\{\cos^2[x(t-\tau_D)+\Phi] - \cos\Phi\}$$

  - feedback loop optical power: β (0 to ca. 3)
  - Huge time delay: τ_D (10s of μs i.e. a few kms of fiber)
  - linear resonant filter: $\frac{\Omega_0}{2\pi} \simeq 10GHz, BW\simeq 80$MHz, $m \simeq 2.5\,10^{-3}$

- **Complex amplitude equation (slowly varying env.).**
  - microwave harmonic asumption: $x(t) = \frac{1}{2}\mathcal{A}(t)\cdot e^{i\Omega_0 t} + c.c.$
  - derived Ikeda-like dynamics for the complex envelope with a Bessel function as

$$\dot{\mathcal{A}}(t) = -\mu\mathcal{A}(t) - 2\mu\gamma\, e^{-i\sigma}\, J_1[2|\mathcal{A}(t-\tau_D)|]\, e^{i\arg\mathcal{A}(t-\tau_D)}$$

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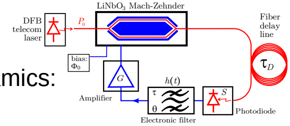

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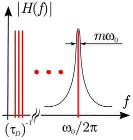

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Phase noise modeling

Predicting the phase noise spectral profile

- Envelope equation including the noise sources

$$\dot{A}(t) = -\mu\mathcal{A}(t) - \mu[1+\eta_{\mathrm{m}}(t)]\mathcal{A}(t-\tau_D) + \xi_{\mathrm{a}}(t)$$

- Derived phase dynamics ($\mu = \Delta\Omega/2 = m\Omega_0$):

$$\dot{\psi}(t) = -\mu[\psi(t) - \psi(t-\tau_D)] + \frac{\mu}{2Q}\eta_{\mathrm{m}}(t) + \frac{\mu}{|\mathcal{A}_0|}\xi_{\mathrm{a},\psi}(t)$$

- Translate the phase dynamics into the Fourier domain

$$|\Psi(\omega)|^2 = \mu^2 \left| \frac{(2Q)^{-1}\tilde{\eta_{\mathrm{m}}}(\omega) + |\mathcal{A}_0\tilde{\xi}_{\mathrm{a},\psi}(\omega)|^{-1}}{i\omega + \mu[1 - e^{-i\omega\tau_D}]} \right|^2$$

(Chembo *et al.*, *IEEE J. Quantum Electron.* 2009).

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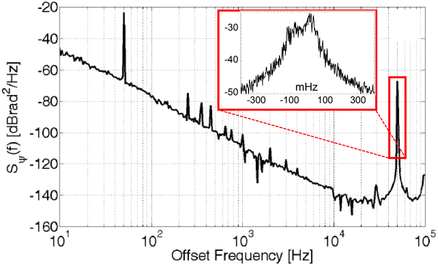
(Chembo *et al.*, *IEEE J. Quantum Electron.* 2009).

Phase noise spectra measurement

Excellent agreement theoretical prediction vs. measure

- Noise floor: -146 dBc/Hz
- Height of the first spurious delay mode peak: 120 dB
- Width at half maximum of the first spurious delay mode peak: 35 mHz

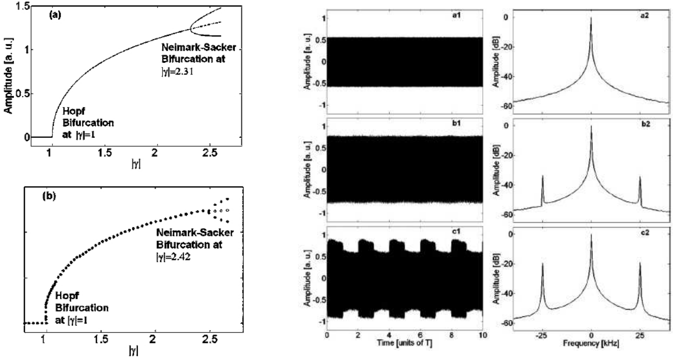


(Volianskiy *et al.*, *IEEE J. Lightwave Technol.* 2010).

Typical nonlinear dynamics OEO instabilities

- **Bifurcation of the microwave delay oscillator**

  Stability analysis: 2nd bifurcation @ $\gamma = 2.3$ with crenelated envelope (first at $\gamma = 1$: microwave oscillation)



(Chembo *et al.*, *Optics Lett.* 2007, Chembo *et al.*, *Optics Express* 2008, Volianskyi *et al.*, *JOSA B* 2008, Chembo *et al.*, *IEEE J. Quantum Electron.* 2008).

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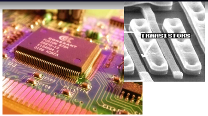
From Neural Networks to RC



- **Neural Network Computing**

  Artificial intelligence, network of coupled oscillators,
  learning, actual demonstration via "conventional computer" simulations

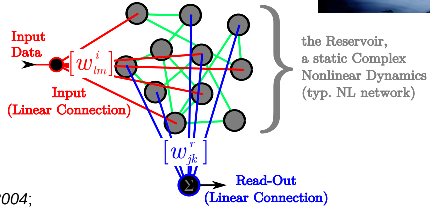
- **Cognitive brain research, bio-inspired computing principles**

  biologic neural network, time trajectories corresponding to pulse train solutions

- **Echo State Network (ESN), Liquid State Machines (LSM), RC**

  Novel architecture exhibiting universal computational potential

- **Basic architecture**

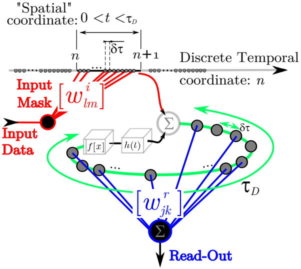


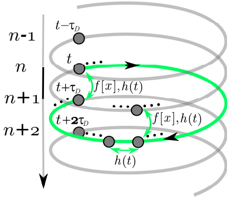
(Jaeger *et al.*, *Techn.Report GMD 2002 & Science 2004*;
Maass, *Neural Comput. 2002*; Jäger and Maass, & Principe, *Neural Networks 2007*).

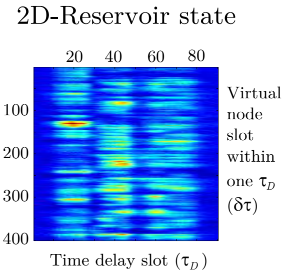
femto-st SCIENCES & TECHNOLOGIES

Delay Dynamics as a Reservoir

- **Spatio-Temporal viewpoint of a DDE**







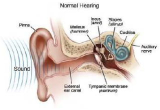
- **Discrete time variable: Delay time step**
- **Virtual Spatial variable: internal delay waveform**

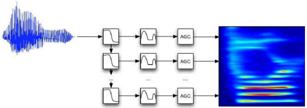
Node network ≡ fine sampling within a delay

Standard test: spoken digit recognition

Data base of 500 Spoken digits, TI46

- Conventional pre-processing





- Processing with the EO nonlinear single delay reservoir

(Appeltant *et al.*, *Nat. Commun.* 2011; Larger *et al.*, *Opt. Expr.* 2012; Paquot *et al.*, *Scient. Rep.* 2012; Martinenghi *et al. Phys. Rev. Lett.* 2012)

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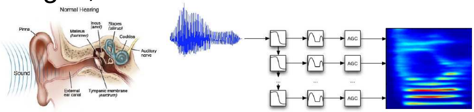

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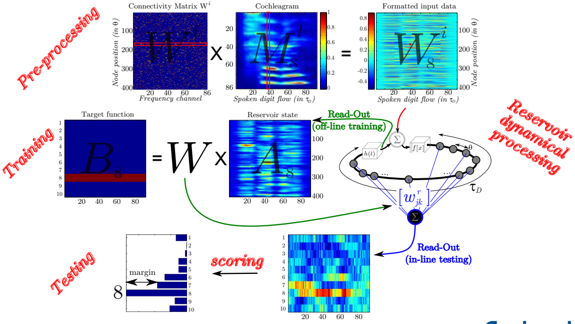
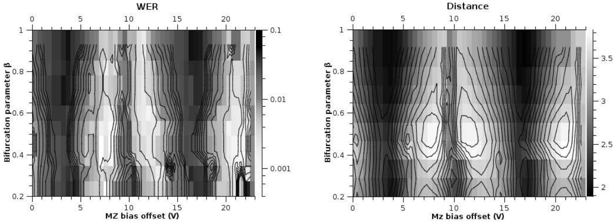


(Appeltant *et al.*, *Nat. Commun.* 2011; Larger *et al.*, *Opt. Expr.* 2012; Paquot *et al.*, *Scient. Rep.* 2012; Martinenghi *et al. Phys. Rev. Lett.* 2012)

First demonstrated photonic LSM

- **Simple Ikeda-like electro-optic architecture**
- **Pre- and post-processing performed externally**
- **Excellent experimental results on a benchmark test**

Spoken Digit Recognition with Word Error Rate $< 0.2\%$ (set size limited, 500 spoken digits data base)



- **Many remaining degrees of freedom for optimization**

Work in progress within the 3 apps, and even more

- **Chaos secure communications**

  hybrid analog / digital emitter / receiver architecture
- **Microwave optoelectronic oscillator**

  Multiple delays feedback, cubic phase fedback term, optical ring resonators topologies,. . .
- **Reservoir Computing**

  Plasticity issues, multiple delayed feedback, mutually coupled nonlinear delay dynamics,. . .
- **Broadband chaos for high speed RNGs**
- **And many theoretical issues in delay dynamics stability, solutions,. . .**

Aknowledgements

SEVENTH FRAMEWORK PROGRAMME

cnrs dépasser les frontières

UFC UNIVERSITÉ DE FRANCHE-COMTÉ

ULB

UIB Universitat de les Illes Balears

CSIC

Vrije Universiteit Brussel

IUF

IFISC

phocus towards a PHOtonic liquid state machine based on delay-CoUpled Systems

femto-st SCIENCES & TECHNOLOGIES