Comment

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Comment on "Quantum-enhanced interferometry with weak thermal light"

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Ref. [1] proposes to increase by photon subtraction the sensitivity of interferometry for thermal light. We prove that this is possible only with a low success rate, rendering the method less efficient than detection of all the photons

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The increase of interferometry sensitivity demonstrated in [1] lies on a photon subtraction obtained by diverting with a beam-splitter a small part (10 %) of the thermal light at one output port of the interferometer, see fig. 1, and heralding: only events where one photon is measured by APD1 are retained.

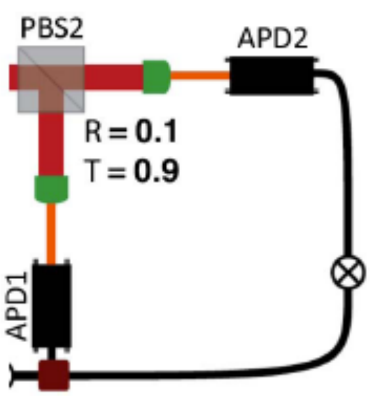

Fig.1: Heralding scheme: measurements on APD2 are retained if and only if a single photon is detected on APD1 (reproduced from [1])

This heralding scheme, named photon subtraction in [1], is claimed to double the mean number of photons measured on APD2. For a photon thermal state after action of an annihilation operator, this is a straightforward result: it is proved in [2] that for every state the average number of photons $\bar{n}_-$ after the action of the annihilation operator is given by:

$$\bar{n}_- = \overline{n^2}/\bar{n} - 1 \qquad (1)$$

where $\bar{n}$ is the mean number of photons in the thermal state corresponding to one temporal mode. The variance of a thermal state is given by:

$$\overline{n^2} - \bar{n}^2 = \bar{n}^2 + \bar{n} \qquad (2)$$

leading to

$$\bar{n}_- = 2\bar{n} \qquad (3)$$

On the one hand, this result is not surprising at low flux. Indeed, the second order coherence function, whose value is 2 for a thermal beam, can be interpreted as a conditional probability. Hence detecting a photon is a rare event, that increases the probability of detecting in the same temporal mode more photons than the mean, because thermal photons are bunched, even at very low fluxes [3]. On the other hand, the doubling in Eq. (3) does not depend on the average flux $\bar{n}$, which is troubling. Indeed, at high flux the probability of detection by APD1 of at least one photon tends to unity and doubling the conditional average means doubling the average. The fact that, most often, more than one photon is detected by APD1 does not improve the plausibility of Eq. (3): it is proved in [1] that the conditional average at APD2 is even higher than $2\bar{n}$ for multiple detections at APD1. Hence, it is necessary to avoid using a model lying on an annihilation operator for the experimental scheme of [1] and to use rather the well-known physics of the beam-splitter. We will see, as foreseen above, that both models are equivalent at very low flux but differ for higher flux.

Let be R=rr* the reflection coefficient of the beam splitter, corresponding to the part of the flux diverted to APD1, and T=tt*=1-R its transmission coefficient. The state of the thermal beam in the mode before the beam splitter can be written as a diagonal density function:

$$\rho_{in} = \sum_{n=0}^{\infty} \frac{\bar{n}^n}{(1+\bar{n})^{n+1}} \,|\, n\rangle\langle n|, \ with \,|\, n\rangle = \frac{\left(a_{in}^{\dagger}\right)^n}{\sqrt{n!}} \,|\, 0_{in}\rangle \qquad (4)$$

The state after the beam-splitter is obtained [4] by applying the same input creation operators on the output modes o1 and o2, corresponding respectively to APD1 and APD2:

$$a_{in} = r\,a_{o1} - t\,a_{o2} \Longrightarrow$$
$$\rho_{out,total} = \sum_{n=0}^{\infty} \frac{\bar{n}^n}{(1+\bar{n})^{n+1}} \mid \psi_{n,out} \rangle \langle \psi_{n,out} \mid$$
$$with \mid \psi_{n,out} \rangle = \frac{\left(r^* a_{o1}^{\dagger} - t^* a_{o2}^{\dagger}\right)^n}{\sqrt{n!}} \mid 0_{o1}, 0_{o2} \rangle \qquad (5)$$

$\mid \psi_{n,out} \rangle$ appears as a coherent superposition. However, we are interested only in the element corresponding to exactly one photon in o1. This not normalized projected wave function reads:

$$\left| \psi_{n,out,reduced} \right\rangle = \frac{n\left(r^*\, t^{*\,n-1}\right)}{\sqrt{n}} \mid 1_{o1}, (n-1)_{o2} \rangle \qquad (6)$$

which gives a normalized diagonal output density operator after heralding :

$$\rho_{out} = \frac{1}{P(1_{o1})} \sum_{n=0}^{\infty} \frac{\bar{n}^n}{(1+\bar{n})^{n+1}}\, n\, R\, T^{n-1} \mid (n-1)_{o2}, 1_{o1}, \rangle \langle 1_{o1}, (n-1)_{o2} \mid$$

(7)

Eq. (7) can be translated in a more intuitive way by using conditional probabilities:

$$P((n-1)_{o2} | 1_{o1}) = \frac{\bar{n}^n}{(1+\bar{n})^{n+1}} n\, R\, T^{n-1} / P(1_{o1}) \qquad (8)$$

In (8) and in all the following P means "probability of" and the vertical bar means "given". The division by $P(1_{o1})$ ensures the normalization of Eq. (7) . $P(1_{o1})$ is therefore given by:

$$P(1_{o1}) = \sum_{n=1}^{\infty} P((n-1)_{o2}\ and\ 1_{o1}) = \sum_{n=1}^{\infty} \frac{\bar{n}^n}{(1+\bar{n})^{n+1}} n\, R\, T^{n-1} \qquad (9)$$

Eq. (8) means that the quantum formalism is rigorously equivalent to a semi-classical model where the conditional probabilities at the output are given by the Bayes theorem:

$$P((n-1)_{o2} | 1_{o1}) = P(n_{in} | 1_{o1}) = \frac{P(1_{o1} | n_{in}).P(n_{in})}{P(1_{o1})} \qquad (10)$$

with

$$P(1_{o1} | n_{in}) = n_{in}\, R\, T^{n_{in}-1}\,,$$
$$P(n_{in}) = \frac{\bar{n}^{n_{in}}}{(1+\bar{n})^{n_{in}+1}} \qquad (11)$$

The first eq. (11) means that there is as many possibilities of obtaining one photon in 1 as the number n_{in} of input photons, each one occurring with a probability $R\,T^{n_{in}-1}$. Because R+T=1, $n_{in}\, R\, T^{n_{in}-1}$ is clearly smaller than 1 whatever n_{in}.

It is now straightforward to calculate all the quantities of interest by using well-known formulae on series. We find:

$$P(1_{o1}) = \frac{\bar{n}\, R}{(1+\bar{n}R)^2} \qquad (12)$$

Note that this result can be established more directly by observing that the number of photons in mode o1 can be viewed as resulting from a random deletion from a Bose-Einstein statistics, hence obeys also a Bose-Einstein statistics with a reduced mean $\bar{n}\, R$ [5]. The conditional mean reads:

$$(\overline{n_{o2}} | 1_{o1}) = \left(\frac{1+\bar{n}R}{1+\bar{n}}\right)^2 \sum_{n=1}^{\infty} \left(\frac{T\bar{n}}{1+\bar{n}}\right)^n (n^2+n) = 2\,\frac{T\bar{n}}{1+\bar{n}R} \qquad (13)$$

For $\bar{n}R << 1$, we retrieve the limit $2\bar{n}$. The one-photon subtraction model appears as correct only in this limit. With the values of ref. 1, R=0.1 and $\bar{n} = 1.14$, we find $(\overline{n_{o2}} | 1_{o1}) = 1.84$, in good agreement with the experimental value of 1.81 given in the supplementary material of [1], and quite far of the value 2 $\bar{n}$ =2.28. We conclude that the discrepancy between this value and the experimental one is not due to loss, as argued in the supplementary material of [1], but rather to the use of a model based on the annihilation operator for a not sufficiently low value of $\bar{n}R$.

A key point is the rate of heralding success. It is argued in [1] that the heralding scheme will have a practical interest if one is able to reduce the subtraction loss to at least 50 %. What is actually relevant is the mean

$$(\overline{n_{o2}}\ and\ 1_{o1}) = (\overline{n_{o2}} | 1_{o1}).P(1_{o1}) = 2\,\frac{RT\bar{n}^2}{(1+\bar{n}R)^3}$$
$$< 2\,\frac{\bar{n}R}{(1+\bar{n}R)^3}\,\bar{n} \qquad (14)$$

The coefficient multiplying $\bar{n}$ in the right hand side of inequality (14) has a maximum value of 0.30, obtained for $\bar{n}R = 0.5$. Hence, the number of "useful photons" cannot be increased beyond $\bar{n}$. This property has a simple physical interpretation: a passive linear beam splitter cannot increase the total number of photons, and, if we do not consider the detector noise, the best strategy is to use all photons without heralding. This issue has been extensively discussed in the context of weak value amplification [6]: selecting the temporal modes with the highest signal-to-noise ratio has an interest only if the detector noise is so high that the signal does not emerge beyond this detector noise for the other modes.

One may wonder about the possibility of using all the temporal modes where at least a click is registered on APD1. Indeed a double click, for example, induces a higher conditional mean than a simple click [1]. Rather than a lengthy calculation of the conditional mean for each number of clicks, we present in the following a simple argument which shows that the result is worse than a non-conditional detection, at least in absence of detector noise. We consider in the following the use at the output of the total number of photons n_o, detected either by APD1 or by APD2. This number is equal to n_{in}, and we can write:

$$\bar{n} = (\overline{n_o} | 0_{o1}) P(0_{o1}) + (\overline{n_o} | (n>0)_{o1}) P((n>0)_{o1}) \qquad (15)$$

leading to:

$$(\overline{n_o}|(n > 0)_{o1}) = \frac{\bar{n} - (\overline{n_o}|0_{o1})P(0_{o1})}{P((n > 0_{o1}))} \quad (16)$$

All the terms in (16) are easy to calculate using series like in Eq. (9) and (13), with the results:

$$(\overline{n_o}|0_{o1}) = \frac{\bar{n}R}{(1+\bar{n}R)}\,,\ P(0_{o1}) = \frac{1}{1+\bar{n}R}\,,$$
$$P((n > 0_{o1})) = 1 - P(0_{o1}) = \frac{\bar{n}R}{1+\bar{n}R} \quad (17)$$

which gives:

$$(\overline{n_o}|(n > 0)_{o1}) = \frac{2\bar{n} + \bar{n}^2 R + 1}{1+\bar{n}R} \quad (18)$$

As expected, the result tends to $2\bar{n} + 1$ if $\bar{n}R << 1$. For higher $\bar{n}R$, the result is smaller than $2\bar{n} + 1$, though greater than the conditional mean for one click $2\frac{T\bar{n}}{1+\bar{n}R} + 1$. Moreover, the conditional mean multiplied by the success rate remains smaller than $\bar{n}$, by construction of Eq. (15).

All these results are valid whatever the quantum efficiency of the photodiodes. Indeed, a non unity quantum efficiency keeps the statistics of n_o Bose-Einstein, with a reduced mean [5]. Then, all the above computations concern the repartition of these detected photons between the photodiodes. Because the detection in a reference experiment without beam-splitter is also performed with the same non-unit quantum efficiency, it can be considered that the total mean photon number $\bar{n}$ refers to a number of detected photons. With this convention, the quantum efficiency has no role in the computations.

We have validated all the above results by generating a million of random numbers obeying a Bose-Einstein distribution. Each of these numbers represents a number of photons at the input and each photon is randomly either reflected with a probability R or transmitted, using random numbers obeying a uniform distribution. Last, they are randomly detected or not, depending on the quantum efficiency. Results are in perfect agreement with the above analytical results.

As quoted in [1], other schemes were proposed in order to subtract exactly a photon from a field. As explained in [7], such a scheme does not correspond to applying an annihilation operator, since the probability of success is ideally unity, whatever the nonzero number of photons in the field. Hence there is no multiplication of the elements of the density operator by the number of photons, unlike for the annihilation operator. This multiplication comes fundamentally from a probability of success proportional to the number of photons. It seems difficult to imagine other schemes that are compatible with this feature than heralding at very low flux. Hence, the above conclusions seem general, and the realization with a beam-splitter seems generic.

To conclude, the increase of the conditional mean obtained by heralding is by principle not sufficient to compensate for the low heralding success rate. Hence, in the absence of a strong detector noise, using all photons is preferable.

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